

**Manipal Institute of Technology**  
**Department of Aeronautical and Automobile Engineering**

AAE 3157 Flight Dynamics  
End Semester Exam - Epad  
06 Jan 2021

Total Marks : 50

Duration: 3 hrs

**Answer All questions**

**Make suitable Assumptions wherever required**

**Draw suitable sketches/figures/plots to illustrate your solution and final answer**

**Q1.a)** A subsonic aircraft is trimmed in level flight at a speed of  $V_1$  at certain altitude  $h_1$  at an angle of attack  $\alpha = 4^\circ$  using an elevator deflection of  $\delta_{e1}$ . The same aircraft with the same CG and weight flying level at a higher altitude of  $h_2 (> h_1)$  needs a higher  $\alpha$  of  $8^\circ$  at a slightly higher flight speed of  $V_2$  and uses an elevator deflection of  $\delta_{e2}$  to trim. For these level flight conditions essentially in subsonic regime, assume that the longitudinal trim curve,  $\delta_e$  Vs  $\alpha$  does not depend on speed/ altitude/Mach number. Draw  $\delta_e$  Vs  $\alpha$  curve for the level flight and obtain a relationship between  $\delta_{e0}$ ,  $\delta_{e1}$  and  $\delta_{e2}$ , where  $\delta_{e0}$  denotes elevator angle intercept value extrapolated for trim at  $\alpha = 0$ .  
(4)

b) For the above aircraft the following data is supplemented to estimate trim elevator required for level flight: Stall angle  $\alpha_{\text{stall}} = 12^\circ$ ,  $\delta_{e0} = +2.5^\circ$  and  $\delta_{e\text{min}} = -20^\circ$ . If the maximum available up elevator ( $\delta_{e\text{min}}$ ) is used for trimming the manoeuvre flight at  $\alpha_{\text{stall}}$ , for the forward most CG and the static margin SM for forward most CG is 0.8MM (Manoeuvre Margin), obtain the elevator angle  $\delta_{e\text{trim}}$  required for level flight at  $\alpha_{\text{stall}}$ . Show your result in the longitudinal trim curve ( $\delta_e$  Vs  $\alpha$ )  
(6)

**Q2. a)** A transport aircraft has 4 engines symmetrically mounted on its wings of 65 m span and area  $525 \text{ m}^2$ . Inboard and outboard engines are located respectively at 11.7 m and 21.7 m from aircraft plane of symmetry. The SL maximum thrust ( $T_{\text{SL}}$ ) per engine is 282 kN. The engine thrust  $T$  at any altitude  $h$  is related to its thrust  $T_{\text{SL}}$  at sea level and the density ratio  $\sigma$  at the altitude  $h$  by  $T = T_{\text{SL}}\sigma$ . The aircraft encounters **starboard** outboard engine failure at  $V_F = 160 \text{ m/s}$  at  $h_F = 2000 \text{ m}$  ( $\sigma_F = 0.8216$ ). The engines were set at 80% throttle for trimmed level flight prior to engine failure. Following the engine failure, Pilot is advised to descend to recommended control altitude  $h_C$  with  $\sigma_C = (4/3)\sigma_F$  (for a 4 engine aircraft) and also reduce the flight speed to recommended control speed  $V_C = [\sqrt{3/4}]V_F$  (also for a 4 engine aircraft), retaining the throttle setting (80%) and fly level at  $h_C$  after the descent. Calculate the yawing moment  $(N_E)_F$  and the yawing moment coefficient  $(C_{NE})_F$  due to asymmetric power, immediately following the engine failure and  $(N_E)_C$  and  $(C_{NE})_C$  after descending to control altitude  $h_C$  and decelerating to control speed  $V_C$ .  
(4)

b) Given the rudder control power  $C_{N\delta_r} = 0.0014/\text{deg}$ , obtain the rudder deflection required to trim asymmetric yawing moment to maintain zero side slip angle at initial altitude  $h_F$  and at the lower altitude  $h_C$  after descent. Show in a plan view of the aircraft, the asymmetric thrust, resulting yawing moment and rudder deflection required to trim the yawing moment and both the altitudes.  
(4)

c) If the aircraft encounters a side gust following the engine failure, which of the following two side gust cases is critical from trimming yawing moment: – side gust from the failed engine side or the side gust from the side where both the engines are working? (2)

**Q3 a)** Following standard notations, the 6 Force and Moment equations governing aircraft motion are given below:

$$\begin{aligned}(-mg \sin \theta) + X &= m (U_{\dot{}} + qW - rV) \\(mg \cos \theta \sin \varphi) + Y &= m (V_{\dot{}} + rU - pW) \\(mg \cos \theta \cos \varphi) + Z &= m (W_{\dot{}} + pV - qU) \\L &= I_x p_{\dot{}} - I_{xz}(pq + r_{\dot{}}) + (I_z - I_y) qr \\M &= I_y q_{\dot{}} + (I_x - I_z) rp + I_{xz}(p^2 - r^2) \\N &= I_z r_{\dot{}} - I_{xz}(p_{\dot{}} - qr) + (I_y - I_x) pq\end{aligned}$$

where (X, Y, Z) and (L, M, N) are aerodynamic forces and moments. Considering dependence of inertia force components on two of the three Euler angles ( $\varphi$ ,  $\theta$ ,  $\psi$ ), as seen above, suggest two additional equations to close the above set of 6 equations for the motion parameters (U, V, W), (p, q, r) and ( $\varphi$ ,  $\theta$ ). Starting from this set of 8 equations, obtain 2 decoupled sets of equations for Longitudinal motion and Lateral – Directional motion of aircraft. State clearly your assumptions made in obtaining decoupled sets of equations. (5)

**b)** The characteristic equation of coupled longitudinal and lateral-directional aircraft motion is known to be an 8<sup>th</sup> order equation. A typical set of 8 roots of such a characteristic equation for some flight condition is given below:

$$\lambda_{1,2} = -4.4 \pm i 65.5, \lambda_3 = -2, \lambda_4 = 0.05, \lambda_{5,6} = -0.35 \pm i 12.5 \text{ and } \lambda_{7,8} = -1.4 \pm i 41.5$$

Show these roots in the  $\lambda$  plane ( $\eta$ ,  $\omega$ ) indicating the nature of the aircraft dynamic modes associated with the 3 pairs of complex conjugate roots- period (short/medium/long) and damping (highly/moderately/lightly) and 2 real roots (highly/negatively damped). Also identify the aircraft modes associated with all the 8 roots including 2 real roots. Obtain time to half  $T_{1/2}$  or time to double  $T_2$  as applicable for all the modes and period T for the periodic modes (5)

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**Q4 a)** A set of 3 linearised Force and Moment equations given below are used for studying lateral – directional dynamics and response of an aircraft

$$\begin{aligned}\Delta \dot{v} &= Y_v \Delta v + Y_p \Delta p + (Y_r - u_0) \Delta r + (g \cos \theta_0) \Delta \phi + Y_{\delta_r} \Delta \delta_r \\ \Delta \dot{p} - (I_{xz}/I_{xx}) \Delta \dot{r} &= L_v \Delta v + L_p \Delta p + L_r \Delta r + L_{\delta_a} \Delta \delta_a + L_{\delta_r} \Delta \delta_r \\ \Delta \dot{r} - (I_{xz}/I_{zz}) \Delta \dot{p} &= N_v \Delta v + N_p \Delta p + N_r \Delta r + N_{\delta_a} \Delta \delta_a + N_{\delta_r} \Delta \delta_r\end{aligned}$$

From this set of equations, choose relevant 2 equations used for studying spiral dynamics and carry out applicable simplifications for capturing essential features of spiral mode of lateral - directional dynamics of the aircraft. Obtain the root(s) of the

characteristic equation for this spiral motion and describe the aircraft motion with suitable sketches. (6)

b). Calculate the time to half or time to double as applicable for the spiral characteristics of the aircraft with the following data:

$$C_{l\beta} = -0.103 \text{ /rad} \quad C_{lr} = 0.11 \text{ /rad} \quad C_{n\beta} = 0.137 \text{ /rad} \quad C_{nr} = -0.16 \text{ /rad.} \quad (4)$$

Q5 a) The transfer function given below is for one of the longitudinal dynamic responses (SPO) of angle of attack of aircraft for elevator control input:

$$[\alpha/\delta_e] = \{-0.746s - 208.6\} / \{675 s^2 + 1361.6 s + 5452.45\}$$

Find the natural frequency and damping ratio for this mode. Using the final value theory or otherwise, find the steady state value of angle of attack  $\alpha$  in response to step elevator input of  $-3^\circ$ . (6)

b) The following three qualitative descriptions for the three important attributes of an aircraft in flight are used in classifying the Handling Qualities of the aircraft:

**Pilot Load** : Excessively High / Moderately High / Acceptable),

**Mission Accomplishments**: Partially accomplished & not acceptable/  
Accomplished with some deterioration / Fully accomplished

**Flight Safety**: Safe/Unsafe,

Using the above descriptions, classify the Aircraft Handling Quality into Levels 1, 2 and 3. What are the dynamic characteristics of the aircraft which affect the pilot in his flying mission and these Handling Quality Levels as well. (4)