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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
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V SEMESTER B. TECH (ELECTRICAL & ELECTRONICS ENGINEERING)
ONLINE EXAMINATIONS, JANUARY - FEBRUARY 2021

DIGITAL SIGNAL PROCESSING [ELE 3152]
REVISED CREDIT SYSTEM

Time: 3 Hours **Date: 30 January 2021** **Max. Marks: 50**

Instructions to Candidates:

- Answer **ALL** the questions.
- Missing data may be suitably assumed.
- Bold and underlined sample represents a sample at zero location.
- * represents convolution sum.
- Use of DSP reference manual is allowed.

1A The discrete time signals $x[n]$ and $h[n]$ are given by $x[n] = [1, \underline{\mathbf{1}}, 1, 1, 1, 1/2]$ and $h[n]$ =[-2, -3/2, -1, -1/2, $\underline{\mathbf{0}}$, 1/2, 1, 3/2, 2]
Sketch the following:
a) $x[n+2]\, h[1-2n]$
b) $h[n]\, x\left[-\frac{n}{3}\right]$ **3**

1B Evaluate the convolution sum of the following discrete time sequences:
$$(u[n+10] - 2u[n-1] + u[n-4]) \text{ and } \beta^n u[n];\ \ |\beta| < 1$$ **4**

1C Use the properties to find the DTFT of
$$x[n] = \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n} * \frac{\sin\left(\frac{\pi}{2}(n-3)\right)}{\pi(n-3)}$$ **3**

2A Obtain the time-domain signal of $X[z] = \frac{1+\frac{7}{6}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)};\quad \frac{1}{3} < |z| < \frac{1}{2}$
Mention whether this system is stable and causal? **5**

2B. If the sampling of a CT signal $x(t) = \cos(\Omega t)$ results in the sequence $x[n] = \cos\left(\frac{\pi n}{5}\right)$ at a sampling rate of 1000 samples/sec, mention any three possible values of Ω that could have resulted in the given sequence $x[n]$. **3**

2C. Find the inverse DFT of the sequence $X[k] = \begin{cases} 5, & k=0 \\ 3, & k=2,8 \\ 2, & otherwise \end{cases}$; where $0 \le k \le 9$.

Obtain the expression for x$[n]$. **2**

3A. Determine the output y[n] of a filter using Overlap-save method whose impulse response is $h[n] = -2\delta[n+1] + \delta[n] - \delta[n-1]$, and the input sequence is $x[n] = 4\cos\left(\frac{n\pi}{4}\right)[u(n) - u(n-7)]$. Take sub-frame length of 4. **3**

3B. Find the 8-point DFT of the signal $x[n] = \begin{cases} 2\sin\left(\frac{n\pi}{2}\right) - |n|\,; & -3 \le n \le 3 \\ 0; & otherwise \end{cases}$.

Use radix-2, DIT-FFT algorithm. Show all the intermediate values on the signal flow graph. ***4***

3C. Design an ideal low-pass filter using frequency sampling method to satisfy the following conditions:

Length of filter = 9

Sampling frequency = 32 KHz

Passband: $0 \le F \le 4$ KHz

Also find the transfer function of the filter. ***3***

4A. Consider the system function $H[z] = \frac{1+3z^{-1}+3z^{-2}+z^{-3}}{1-0.9z^{-1}+064z^{-2}-0.58z^{-3}}$. Sketch the lattice-ladder realization of the system. Mention whether the system is stable. ***6***

4B. Given a composite signal $x[n] = \cos 0.1n + \cos 0.4n$. For a specific application it is required to reject the signal of low frequency and to retain the signal of high frequency. The impulse response of the filter is causal and is given by $h[n] = \{\underline{\boldsymbol{\alpha}}, \beta, \alpha\}$. Find the impulse response. ***4***

5A. Using windowing technique design a symmetric, 11-tap FIR filter with desired frequency response:

$$H_d[e^{j\omega}] = \begin{cases} 0; & |\omega| < \frac{\pi}{3} \\ 1; & \frac{\pi}{3} \le |\omega| \le \pi \end{cases}$$

Use Hamming window. Draw an appropriate filter structure and also obtain its frequency response. ***5***

5B. A digital lowpass Butterworth filter is required to meet the following specifications:

$$0.8 \le |H[e^{j\omega}]| \le 1; \quad 0 \le \omega \le 0.2\pi$$
$$|H[e^{j\omega}]| \le 0.2; \quad 0.6\pi \le \omega \le \pi$$

Design the filter using bilinear transformation technique. ***5***

DSP - Quick Reference Table

(For ELE 304: Digital Signal Processing)

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MAY 2007

Z-Transform of Basic Signals

Definition: $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$ $\qquad x[n] = \frac{1}{2\pi j} \oint X(z) z^{n-1} dz$

	Signal	Transform	ROC
1.	$\delta[n]$	1	All z
2.	$\delta[n-m]$	z^{-m}	All z except 0 if m> 0 or ∞ if m<0
3.	$u[n]$	$\frac{z}{z-1}$	$\|z\| > 1$
4.	$-u[-n-1]$	$\frac{z}{z-1}$	$\|z\| < 1$
5.	$a^n u[n]$	$\frac{z}{z-a}$	$\|z\| > a$
6.	$-a^n u[-n-1]$	$\frac{z}{z-a}$	$\|z\| < a$
7.	$(n+1)a^n u[n]$	$\frac{z^2}{(z-a)^2}$	$\|z\| > a$
8.	$\sin \omega_0 n \, u[n]$	$\frac{z \sin \omega_0}{z^2 - 2z \cos \omega_0 + 1}$	$\|z\| > 1$
9.	$\cos \omega_0 n \, u[n]$	$\frac{z(z - \cos \omega_0)}{z^2 - 2z \cos \omega_0 + 1}$	$\|z\| > 1$

Properties of z-Transform

Property	Signal	z-Transform
	$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$
1. Linearity	$a_1x_1[n] + a_2x_2[n]$	$a_1X_1(z) + a_2X_2(z)$
2. Time shifting	$x[n \pm k]$	$z^{\pm k}X(z)$; $x[n]$ is two - sided
	$x[n+k]u[k]$	$z^kX(z) - z^k \sum_{m=0}^{k-1} x[m]z^{-m}$; $x[n]$ is right - sided
3. Time expansion	$x[n/m]$; n Multiple of m	$X(z^m)$
4. Scaling in the z - domain	$a^n x[n]$	$X(z/a)$
5. Time Reversal	$x[-n]$	$X(z^{-1})$
6. Differentiation in the z – domain	$nx[n]$	$-z\frac{d}{dz}X(z)$
7. Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$
8. Conjugation	$x^*[n]$	$X^*(z^*)$

Discrete-Time Fourier Series

Definition: $x[n] = \sum_{k=<N>} X[k]\, e^{jk\omega_0 n}$ $\quad X[k] = \frac{1}{N}\sum_{n=<N>} x[n]\, e^{-jk\omega_0 n}$

$\omega_0 = 2\pi/N$ $\quad N = \textit{fundamental period}$

Properties of DTFS

Property	Signal	DTFS coefficient
	x[n] y[n]	X[k] Y[k]
1. Linearity	Ax[n]+By[n]	AX[k]+BY[k]
2. Time shifting	x[n-m]	$e^{-jk\omega_0 m} X[k]$
3. Frequency shifting	$e^{jM\omega_0 n}$ x[n]	X[k - M]
4. Time expansion	$x[n/m]$; n is multiple of m	$\frac{1}{m}$ X[k] ; period mN
5. Time Reversal	x[-n]	X[−k]
6. Modulation	x[n]y[n]	$\sum_{m=<N>} X[m]Y[k-m]$
7. Periodic Convolution	$x[n] \circledast y[n]$	$N a_k b_k$
8. Conjugation	x*[n]	X*[-k]

$$x[n] \circledast y[n] = \sum_{m=<N>} x[m]\, y[n-m]$$

Parsevaal's Relation $\quad P = \frac{1}{N}\sum_{n=<N>} |x[n]|^2 = \sum_{k=<N>} |X[k]|^2$

Discrete-Time Fourier Transform

Definition: $x[n]=\frac{1}{2\pi}\int_{<2\pi>} X(e^{j\omega})\, e^{j\omega n}\, d\omega \qquad X(e^{j\omega})=\sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$

DTFT of Basic Signals

Signal	Transform
1) $\delta[n]$	1
2) $a^n u[n]; \lvert a\rvert<1$	$\frac{1}{1-a e^{-j\omega}}$
3) $(n+1)\, a^n\, u[n]; \lvert a\rvert<1$	$\left(\frac{1}{1-a e^{-j\omega}}\right)^2$
4) $x[n]=\begin{cases}1; \lvert n\rvert\le M\\ 0; \lvert n\rvert>M\end{cases}$	$\frac{sin((2M+1)\omega/2)}{\sin(\omega/2)}$
5) $\frac{\sin Mn}{\pi n}; 0<M<\pi$	$X(e^{j\omega})=\begin{cases}1; 0\le\lvert\omega\rvert\le M\\ 0; M<\lvert\omega\rvert\le\pi\end{cases}$ *Periodic with* $\omega=2\pi$
6) $\sum_{k=\langle N\rangle} X[k]\, e^{jk\omega_0 n}\ ; \omega_0=\frac{2\pi}{N}$	$2\pi\sum_{k=-\infty}^{\infty} X[k]\, \delta[\omega-k\omega_0]$ X[k] DTFS *coefficient*

Properties of DTFT

Property	Signal	Transform
	$x[n]$	$X(e^{j\omega})$
	$y[n]$	$Y(e^{j\omega})$
1. Linearity	$Ax[n]+By[n]$	$AX(e^{j\omega})+BY(e^{j\omega})$
2. Time shifting	$x[n-m]$	$e^{-j\omega m}X(e^{j\omega})$
3. Frequency shifting	$e^{j\lambda n}x[n]$	$X(e^{j(\omega-\lambda)})$
4. Time expansion	$x[n/m]$; n is multiple of m	$X(e^{j\omega m})$
5. Time reversal	$x[-n]$	$X(e^{-j\omega})$
6. Differentiation in frequency	$n\,x[n]$	$j\frac{d}{d\omega}X(e^{j\omega})$
7. Convolution	$x[n]*y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
8. Modulation	$x[n]y[n]$	$\frac{1}{2\pi}\{X(e^{j\omega})\circledast Y(e^{j\omega})\}$
9. Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$

Note: $\circledast$ indicates periodic convolution

$$X(e^{j\omega})\circledast Y(e^{j\omega}) = \int_{<2\pi>} X\left(e^{j\lambda}\right)Y\left(e^{j(\omega-\lambda)}\right)d\lambda$$

Parsevaal's Relation $$E=\sum_{n=-\infty}^{\infty}|x[n]|^2 = \frac{1}{2\pi}\int_{<2\pi>}\left|X(e^{j\omega})\right|^2 d\omega$$

Discrete Fourier Transform

Definition:

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi kn}{N}} \qquad ; n=0,1,2\ldots\ldots\ldots N-1$$

$$X[k] = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}} \qquad ; \quad k=0,1,2\ldots\ldots\ldots N-1$$

Where N is the number of samples in the frequency domain in the interval (0 to 2π)

Properties of DFT

Property	Time domain Signal $x(n)$ & $y(n)$	Frequency domain Signal $X(k)$ & $Y(k)$
1) Periodicity	$x(n)=x(n+N)$ for all n	$X(k)=X(k+N)$ for all k
2) Linearity	$Ax(n)+By(n)$	$AX(k)+BY(k)$
3) Time reversal	$x(N-n)$	$X(N-k)$
4) Circular time shift	$x((n-l))_N$	$X(k)e^{-j2\pi kl/N}$
5) Circular frequency shift	$x(n)e^{j2\pi ln/N}$	$X((k-l))_N$
6) Complex conjugate	$x^*(n)$	$X^*(N-k)$
7) Circular convolution	$x(n)$ Ⓝ $y(n)$	$X(k)Y(k)$
8) Circular correlation	$x(n)$ Ⓝ $y^*(-n)$	$X(k)\,Y^*(k)$
9) Multiplication of two sequences	$x(n)y(n)$	$\frac{1}{N}X(k)$ Ⓝ $Y(k)$
10) Parsevaal's Theorem	$\sum_{n=0}^{N-1} x(n)y^*(n)$	$\frac{1}{N}\sum_{k=0}^{N-1} X(k)Y^*(k)$

N-Point DFT of Certain Basic Signals

	Time-domain signal	Transform
1.	$\delta(n)$	1
2.	$\delta(n-n_0)$	$e^{-j\frac{2\pi}{N}n_0 k}$
3.	$1; 0 \le n < N$	$N\ \delta(k)$
4.	$1; 0 \le n < L; L \le N$	$e^{-j\frac{2\pi}{N}Lk}\ \dfrac{\sin(\frac{\pi}{N}Lk)}{\sin(\frac{\pi}{N}k)}$
5.	$(-1)^n;\ 0 \le n < N; N\ Even$	$N\,\delta(k-\frac{N}{2})$
6.	$\alpha^n\ ;\ \ 0 \le n < N$	$\dfrac{1-\alpha^N}{1-\alpha e^{-j\frac{2\pi}{N}k}}$

Filter Structure Conversion

Direct Form to Lattice $$A_{m-1}(z)=\frac{A_m(z)-k_m z^{-m}A_m(z^{-1})}{1-K_m^2}$$

Lattice to Direct Form $$A_m(z)=A_{m-1}(z)+k_m z^{-m}A_{m-1}(z^{-1})$$

System Function $$A_m(z)=\frac{F_m(z)}{F_0(z)}=\frac{F_m(z)}{X(z)}$$

Backward System Function $$B_m(z)=\frac{G_m(z)}{G_0(z)}=\frac{G_m(z)}{X(z)}$$

$$B_m(z)=z^{-m}A_m(z^{-1})$$

Lattice-Ladder Structure : Ladder Coefficients

$$H(z)=\frac{C_M(z)}{A_N(z)}$$ $$C_M(z)=c_0+c_1z^{-1}+c_2z^{-2}+-----+c_Mz^{-M}$$ *Polynomial of degree M*

$$C_M(z)=\sum_{k=0}^{M-1}v_k\,z^{-k}A_k(z^{-1})+v_M\,z^{-M}A_M(z^{-1}) \quad ; \; v_M=c_M$$

Frequency Response of FIR Linear Phase Filters

N odd **Symmetric Characteristics**

Pseudo Magnitude $H_1(\omega)=h(\frac{N-1}{2})+2\sum_{n=0}^{(N-3)/2} h(n)\cos(\frac{N-1}{2}-n)\omega$

Phase $\phi(\omega)=\begin{cases} -\frac{N-1}{2}\omega\ ; H_1(\omega)>0 \\ \pi-\frac{N-1}{2}\omega\ ; H_1(\omega)<0 \end{cases}$

N odd **Antisymmetric Characteristics**

Pseudo Magnitude $H_1(\omega)=2\sum_{n=0}^{(N-3)/2} h(n)\sin(\frac{N-1}{2}-n)\omega$

Phase $\phi(\omega)=\begin{cases} \frac{\pi}{2}-\frac{N-1}{2}\omega\ ; H_1(\omega)>0 \\ \frac{3\pi}{2}-\frac{N-1}{2}\omega\ ; H_1(\omega)<0 \end{cases}$

N Even **Symmetric Characteristics**

Pseudo Magnitude $H_1(\omega)=2\sum_{n=0}^{(\frac{N}{2}-1)} h(n)\cos(\frac{N-1}{2}-n)\omega$

Phase $\phi(\omega)=\begin{cases} -\frac{N-1}{2}\omega\ ; H_1(\omega)>0 \\ \pi-\frac{N-1}{2}\omega\ ; H_1(\omega)<0 \end{cases}$

N Even **Antisymmetric Characteristics**

Pseudo Magnitude $H_1(\omega)=2\sum_{n=0}^{(\frac{N}{2}-1)} h(n)\sin(\frac{N-1}{2}-n)\omega$

Phase $\phi(\omega)=\begin{cases} \frac{\pi}{2}-\frac{N-1}{2}\omega\ ; H_1(\omega)>0 \\ \frac{3\pi}{2}-\frac{N-1}{2}\omega\ ; H_1(\omega)<0 \end{cases}$

Window Functions for FIR Filter Design

Name of window	*Window Function*
1. *Rectangular*	$w(n)=\begin{cases}1; 0\le n\le N-1\\ 0; otherwise.\end{cases}$ $N \rightarrow$ length.
2. *Bartlet (Triangular)*	$w(n)=1-\dfrac{2\left\lvert n-\frac{N-1}{2}\right\rvert}{N-1}$
3. *Hanning*	$w(n)=0.5\left(1-\cos\dfrac{2\pi n}{N-1}\right)$
4. *Hamming*	$w(n)=0.54-0.46\cos\dfrac{2\pi n}{N-1}$
5. *Blackman*	$w(n)=0.42-0.5\cos\dfrac{2\pi n}{N-1}+0.08\cos\dfrac{4\pi n}{N-1}$

Frequency-Domain Characteristics of Certain window Functions

Window	**Side-lobe Amplitude, dB**	**Transition width** $\omega_S - \omega_P$	**Stop band Attenuation, dB**
Rectangular	-13	4π/N	-21
Bartlet	-27	8π/N	-25
Hanning	-32	8π/N	-44
Hamming	-43	8π/N	-53
Blackman	-58	12π/N	-74

Useful Impulse – Invariant Transformations

$s-domain$	$z-domain$
$\dfrac{1}{s+p_i}$	$\dfrac{1}{1-e^{-p_iT}\,z^{-1}}$
$\dfrac{1}{(s+p_i)^2}$	$\dfrac{-d}{dp_i}\dfrac{1}{1-e^{-p_iT}\,z^{-1}}$
$\dfrac{b}{(s+a)^2+b^2}$	$\dfrac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$
$\dfrac{s+a}{(s+a)^2+b^2}$	$\dfrac{e^{-aT}(\sin bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$

Butterworth Low Pass Filter

$$|H(j\Omega)|^2=\frac{1}{1+\left(\dfrac{\Omega}{\Omega_C}\right)^{2N}}$$

Butterworth Polynomials

Order N	$B_N(s)$
1.	$s+1$
2.	$s^2+\sqrt{2}\,s+1$
3.	$(s+1)(s^2+s+1)$
4.	$(s^2+0.7654s+1)(s^2+1.8478s+1)$
5.	$(s+1)(s^2+0.618s+1)(s^2+1.618s+1)$

Chebyshev Low Pass Filter

$$|H(j\Omega)|^2 = \frac{1}{1+\varepsilon^2 T_N^2(\Omega/\Omega_p)}$$

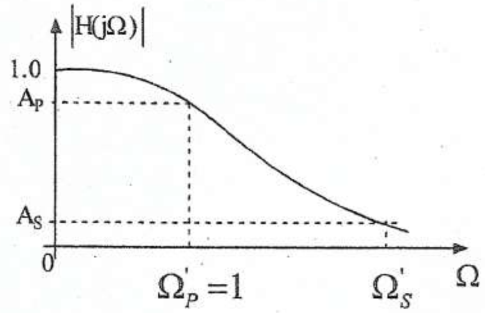
Chebyshev Polynomials

$$T_N(x) = \begin{cases} \cos(N\cos^{-1}x); |x| \le 1 \\ \cosh(N\cosh^{-1}x); |x| > 1 \end{cases}$$

$$T_N(x) = 2x\,T_{N-1}(x) - T_{N-2}(x)$$

Order	$T_N(x)$
0	1
1	x
2	$2x^2-1$
3	$4x^3-3x$
4	$8x^4-8x^2+1$
5	$16x^5-20x^3+5x$
6	$32x^6-48x^4+18x^2-1$

Specifications for Prototype Low Pass Filter



Prototype Low Pass Filter

Type	Specifications for Prototype
High Pass	$\Omega_P' = 1 \quad \Omega_S' = \frac{\Omega_P}{\Omega_S}$
Band Pass	$\Omega_P' = 1 \qquad \Omega_S' = \min(\lvert A\rvert, \lvert B\rvert)$
	$A = \frac{\Omega_{S1}^2 - \Omega_{p1}\Omega_{p2}}{\Omega_{S1}(\Omega_{p2} - \Omega_{p1})}; B = \frac{\Omega_{S2}^2 - \Omega_{p1}\Omega_{p2}}{\Omega_{S2}(\Omega_{p2} - \Omega_{p1})}$
Band Stop	$\Omega_P' = 1 \qquad \Omega_S' = \min(\lvert A\rvert, \lvert B\rvert);$
	$A = \frac{\Omega_{S1}(\Omega_{p2} - \Omega_{p1})}{\Omega_{S1}^2 - \Omega_{p1}\Omega_{p2}}; B = \frac{\Omega_{S2}(\Omega_{p2} - \Omega_{p1})}{\Omega_{S2}^2 - \Omega_{p1}\Omega_{p2}}$
Ω_{p1}, Ω_{p2}	*lower and upper pass band edge frequencies.*
Ω_{s1}, Ω_{s2}	*lower and upper stop band edge frequencies.*

Frequency Transformations from Low Pass Prototype to Other Types

Type	Transformation	Remarks
Low Pass to Low Pass	$s \rightarrow \frac{\Omega'_P}{\Omega_P} s$	Ω'_P – *Pass band edge frequency of prototype* $= 1$ Ω_P – *Pass band edge frequency of new filter*
Low Pass to High Pass	$s \rightarrow \frac{\Omega'_P \Omega_P}{s}$	
Low Pass to Band Pass	$s \rightarrow \Omega'_P \frac{(s^2 + \Omega_{p1}\Omega_{p2})}{s(\Omega_{p2} - \Omega_{p1})}$	
Low Pass to Band Stop	$s \rightarrow \Omega'_P \frac{s(\Omega_{p2} - \Omega_{p1})}{(s^2 + \Omega_{p1}\Omega_{p2})}$	

Time domain and frequency domain relationships for sampled signals.