MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

VII SEMESTER B.TECH. PROGRAM ELECTIVE
MINOR IN MATHEMATICS (Common to all branches)
END SEMESTER EXAMINATION DECEMBER 2020
Subject: Computational Probability & Design Of Experiments [MAT 4013]

Date of Exam: **30/12/2020**   Time of Exam: **09.00 a. m. – 12.00 noon.**   Max. Marks: **50**

Answer ALL the questions. Use of Statistical tables is permitted.

1. **A.** Below is the data from an experiment which was performed at a pulp mill. Plant performance is based on pulp brightness as measured by a reflectance meter. Each of the four shift operators A, B, C and D made five pulp hand sheets from unbleached pulp. Reflectance was read for each of the hand sheets using a brightness tester, as reported in the tabulated data. A goal of the experiment is to determine whether there are differences between the operators in making hand sheets and reading their brightness.

Operator			
A	**B**	**C**	**D**
59.8	59.8	60.7	61.0
60.0	60.2	60.7	60.8
60.8	60.4	60.5	60.6
60.8	59.9	60.9	60.5
59.8	60.0	60.3	60.5

Analyze the data at 5% level of significance, $(F(3,16) = 3.24)$, to assist the investigator in order to draw proper inference. **(7 marks)**

B. Write the incidence matrix for the design having nine treatments and twelve blocks.

Blocks	Treatments			Blocks	Treatments		
1	1	2	3	7	1	6	8
2	4	5	6	8	2	4	9
3	7	8	9	9	3	5	7
4	1	4	7	10	1	5	9
5	2	5	8	11	2	6	7
6	3	6	9	12	3	4	8

(3 marks)

2. **A**. An experiment was carried out to determine the effect of claying the ground on the field of barley grains. The amount of clay used were as follows.

A: no clay   B: clay at 100 per acre   C: clay at 200 per acre   D: clay at 300 per acre
The yield were in plots of 8 mts by 8mts and layout was

Rows	Columns			
	I	II	III	IV
I	D 29.1	B 18.9	C 29.4	A 5.7
II	C 16.4	A 10.2	D 21.2	B 19.1
III	A 5.4	D 38.8	B 24.0	C 37.0
IV	B 24.9	C 41.7	A 9.5	D 28.9

Stating the assumptions along with the model, write the hypothesis for testing the significance of the row effects, column effects and the treatment effects, and analyze the data. Test at 5% level of significance. Table value of F(3,6) = 4.76 at 5% level of significance. **(7 marks)**

B. Check whether the design is a balanced incomplete block design. **(3 marks)**

Blocks	Treatments	
1	1	2
2	1	3
3	1	4
4	2	3
5	2	4
6	3	4

3. Estimate the missing observations and analyze the data corresponding to the experiment of testing the strength index of cotton, after specifying the model. **(10 marks)**

Treatments (pounds of potassium oxide/acre)	Blocks		
	1	2	3
36	u_1	8.00	7.93
54	8.14	8.15	7.87
72	7.76	u_2	7.74
108	7.17	7.57	7.80
144	7.46	7.68	7.21

4. **A**. Assume that the survival times follow the distribution having density

$$p_\theta(x) = \frac{2x}{\theta} e^{\frac{-x^2}{\theta}}, \theta > 0, x > 0.$$ The data below refers to survival times of patients (in months) using two drugs. Use likelihood ratio test (LRT) to test the hypothesis that the two drugs are equally effective.

Drug A: 13, 12, 6, 4, 7, 15, 9, 10, 17, 18
Drug B: 14, 12, 10, 18, 17, 16, 14, 12

(5 marks)

B. The yield of a chemical process is being studied. The two most important variables are thought to be pressure and temperature. Two levels of each factor are selected, and a factorial experiment with 2 replicates is performed. The yield data is given in the following table below. Analyse the data and draw conclusion. Take $\alpha = .05$.
F(1,3)=10.13, F(3,3)=9.28, F(1,7)=5.59, F(3,7)=4.35 **(5 marks)**

Temperature (^{o}C)	Pressure (psi) 200	215
150	90.4	90.7
	90.2	90.6
160	90.1	90.5
	90.3	90.6

5. **A.** The following table gives the layout and the results of a factorial design laid out in four blocks. The purpose of the experiment is to determine the effect of the three kinds of fertilizers namely, Nitrogen N, Potash K and Phosphate P on potato crop yield.

Block 1	nk	kp	p	np	1	k	n	nkp
	291	391	312	373	101	265	106	450
Block 2	kp	p	k	nk	n	nkp	np	1
	407	324	272	306	89	449	338	106
Block 3	p	1	np	kp	nk	k	n	nkp
	323	87	324	423	334	279	128	471
Block 4	np	nk	n	p	k	1	nkp	kp
	361	272	103	324	302	131	437	435

Compute the factorial effects total using Yates' method.

B. The following plan corresponds to a 2^3 factorial experiment replicated four times.

	Replicate 1		Replicate 2		Replicate 3		Replicate 4	
Blocks	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
	abc	ab	abc	ac	abc	ab	abc	ab
	a	ac	ab	bc	bc	ac	ac	bc
	b	bc	c	a	a	b	b	a
	c	(1)	(1)	b	(1)	c	(1)	c

Identify the factorial effects, if any, which are confounded. **(6+4)**

****