ECE 2154 NETWORK ANALYSIS (E&C)

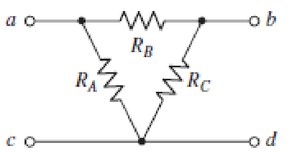
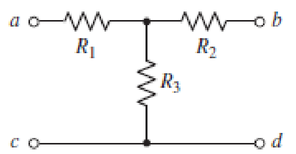
Symbols and Units

Quantity	Symbols	Unit	Equivalent Unit	Abbreviation
Charge	q	coulomb	-	C
Current	i, I	ampere	coulomb/Second	amp
Flux Linkages	ψ	weber-turn	-	Wb
Energy	w, W	Joul	newton-meter	J
Voltage	v, V	Volt	Joul/coulomb	V
Power	p, P	Watt	Joul/second	W
Capacitance	C	Farad	Coulomb/volt	F
Inductance	L, M	Henry	Weber/ampere	H
Resistance	R	Ohm	volt/ampere	Ω
Conductance	G	Mho	ampere/volt	℧
Time	t	second	-	sec
Frequency	f	Hertz	cycles/second	H_Z
Frequency	ω	Radian/second	$\omega = 2\pi f$	-

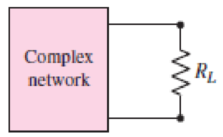
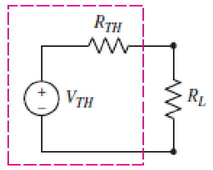
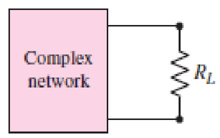
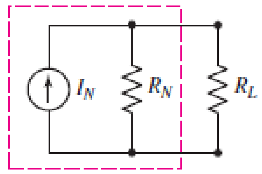
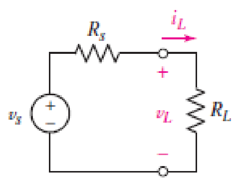
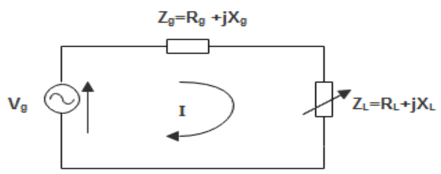
Relationships for the parameters

Parameter	Basic Relationship	Voltage-Current Relationship		Energy
R $G = \frac{1}{R}$	$v = \mathrm{R}i$	$v_R = \mathrm{R}i_R$	$i_R = \mathrm{G}v_R$	$w_R = \int v_R i_R \mathrm{dt}$
L(or M)	$\psi = \mathrm{L}i$	$v_L = L\frac{\mathrm{d}i_L}{\mathrm{dt}}$	$i_L = \frac{1}{L}\int v_L \mathrm{dt}$	$w_L = \frac{1}{2}Li^2$
C $D = \frac{1}{C}$	$q = \mathrm{C}v$	$v_c = \frac{1}{C}\int i_C \mathrm{dt}$	$i_C = C\frac{\mathrm{d}v_C}{\mathrm{dt}}$	$w_c = \frac{1}{2}\mathrm{C}v^2$

Star – Delta Conversion:

Start - delta (wye-delta) conversion	Star to delta conversion: $R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$ $R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$ $R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$ Delta to star conversion: $R_1 = \frac{R_AR_B}{R_A + R_B + R_C}$ $R_2 = \frac{R_BR_C}{R_A + R_B + R_C}$ $R_3 = \frac{R_AR_C}{R_A + R_B + R_C}$	Delta Connection  Star Connection 

Network Theorems:

Thevenin's Theorem	$$R_{TH} = \frac{V_{OC}}{I_{SC}}$$ Where, V_{OC} : Open circuit voltage across the terminals (Thevenin's voltage V_{TH}) I_{SC} : Current through the short circuited terminals (Norton's current I_N) R_{TH} : Thevenin's equivalent resistance R_N : Norton's equivalent resistance	.  
Norton's Theorem	$$R_N = \frac{V_{OC}}{I_{SC}}$$ Where, V_{OC} : Open circuit voltage across the terminals (Thevenin's voltage V_{TH}) I_{SC} : Current through the short circuited terminals (Norton's current I_N) R_{TH} : Thevenin's equivalent resistance R_N : Norton's equivalent resistance	 
Millman's Theorem 1) For n number of parallel connected voltage sources with internal impedances 2) For n number of series connected current sources with shunt impedances	$$E = \frac{\sum_{i=1}^{n} E_i Y_i}{\sum_{i=1}^{n} Y_i} \quad \text{in series with } Z = \frac{1}{\sum_{i=1}^{n} Y_i}$$ $$I = \frac{\sum_{i=1}^{n} \frac{I_i}{Y_i}}{\sum_{i=1}^{n} \frac{1}{Y_i}} \quad \text{in shunt with } Y = \frac{1}{\sum_{i=1}^{n} \frac{1}{Y_i}}$$	
Tellegen's Theorem	If two networks are having the same graph with different elements, then, $$\sum_{k=1}^{b} V_{kN1} \times i_{kN1} = 0, \quad \sum_{k=1}^{b} V_{kN2} \times i_{kN2} = 0$$ And $\sum_{k=1}^{b} V_{kN2} \times i_{kN1} = 0, \quad \sum_{k=1}^{b} V_{kN1} \times i_{kN2} = 0$	
Maximum Power Transfer theorem	 	

Case	Details	
1	DC circuit	R_L=R_{th} $P_{max} = \frac{V_{TH}{}^2}{4R_{TH}}$
2	AC circuit with both load resistance and reactance are variable	$Z_L = Z_g^*$ i.e., $R_L = R_g, X_L = -X_g$
3	AC circuit with load being purely resistive	$R_L = \lvert Z_g \rvert = \sqrt{R_{g^2} + X_{g^2}}$
4	AC circuit with load being variable resistance and fixed reactance	$R_L = \lvert Z_g + jX_L \rvert = \sqrt{R_g^2 + (X_g + X_L)^2}$

Coupled Circuits.

(i) Self Inductance

$$L = N\frac{d\varphi}{\mathrm{di}} \quad \text{and in air L} = \frac{N\varphi}{i}$$

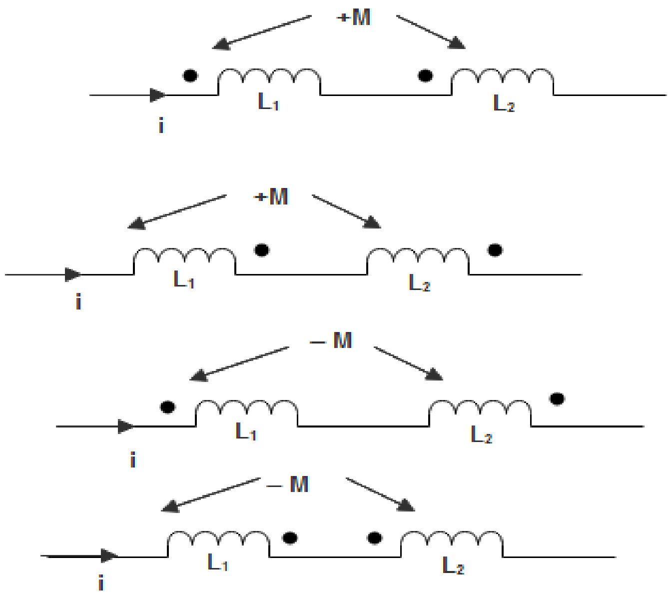
$$v_L = L\frac{\mathrm{di}}{\mathrm{dt}} = N\frac{d\varphi}{\mathrm{dt}}$$

(ii) Mutual Inductance

$$M = N_1\frac{d\varphi_{21}}{\mathrm{di}_2} = N_2\frac{d\varphi_{12}}{\mathrm{di}_1} \quad \text{and in air M} = N_1\frac{\varphi_{21}}{i_2} = N_2\frac{\varphi_{12}}{i_1}$$

$$\text{Also} \quad M = k\sqrt{L_1 \times L_2}$$

(iii) Dot Convention in Coupled Circuits.



Series RLC Resonant Circuit:

Resonant frequency $f_0 = \frac{1}{2\pi\sqrt{\mathrm{LC}}}$

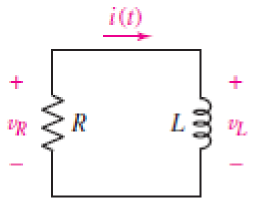
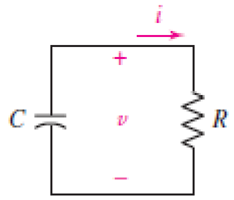
Quality Factor $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 \mathrm{CR}} = \frac{1}{R}\sqrt{\frac{L}{C}}$

Bandwidth $B = \frac{f_0}{Q} = \frac{1}{2\pi} \times \frac{R}{L}$
Impedance of the circuit $Z = R \times [1 + j2Q\delta]$

Parallel Resonant Circuit.

Anti resonant frequency, $f_{\text{ar}} = \frac{1}{2\pi} \times \sqrt{\frac{1}{\text{LC}}} \times \sqrt{1 - \frac{R^2 C}{L}}$
Dynamic resistance,$Z_{\text{ar}} = R_{\text{ar}} = \frac{L}{\text{RC}}$
Impedance of the circuit, $Z = \frac{R_{\text{ar}}}{1 + j2Q\delta}$

Transient Analysis:

Source free RL circuit	$\frac{di}{dt} + \frac{R}{L} i = 0$ $i(t) = I_0 e^{\frac{-Rt}{L}}$ where, I_0 is the initial current at time $t = 0$	
Source free RC circuit	$\frac{dv}{dt} + \frac{v}{RC} = 0$ $v(t) = v(0) e^{\frac{-t}{RC}}$ v_0 is the initial voltage across capacitor at time $t = 0$	
Solution of standard first order differential equation	$\frac{di}{dt} + Pi = Q$ $i = e^{-Pt} \int Q\, e^{Pt} dt + Ae^{-Pt}$	Total response = Forced response + Natural response
Source free Parallel RLC circuit	$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0$ Characteristic Equation: $s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0$ Roots: $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - w_0{}^2}$ Where $\alpha = \frac{1}{2RC}$, $w_0 = \frac{1}{\sqrt{LC}}$	Natural Response
Source free series RLC circuit	$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$ Characteristic Equation: $s^2 + \frac{R}{L} s + \frac{1}{LC} = 0$ Roots: $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - w_0{}^2}$ Where $\alpha = \frac{R}{2L}$, $w_0 = \frac{1}{\sqrt{LC}}$	Natural Response

Laplace Transforms:

Laplace Transform	$F(s) = \int_0^{+\infty} f(t)\, e^{-st} dt$
Inverse Laplace Transform	$f(t) = L^{-1} F(s)$

Table of Laplace Transforms

$f(t)$	$F(s) = \int f(t)e^{-\mathrm{st}}\,\mathrm{dt}$
$u(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$\frac{t^{n-1}}{(n-1)!}$ n is a integer	$\frac{1}{s^n}$
e^{at}	$\frac{1}{s-a}$
$e^{-\mathrm{at}}$	$\frac{1}{s+a}$
$\mathrm{te}^{\mathrm{at}}$	$\frac{1}{(s-a)^2}$
$\mathrm{te}^{-\mathrm{at}}$	$\frac{1}{(s+a)^2}$
$\frac{1}{(n-1)}t^{n-1}e^{\mathrm{at}}$	$\frac{1}{(s-a)^n}$
$\frac{1}{(n-1)}t^{n-1}e^{-\mathrm{at}}$	$\frac{1}{(s+a)^n}$
$1-e^{\mathrm{at}}$	$\frac{-a}{s(s-a)}$
$\frac{1}{\omega}\sin\omega t$	$\frac{1}{s^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
$1-\cos\omega t$	$\frac{\omega^2}{s(s^2+\omega^2)}$
$\sin(\omega t+\theta)$	$\frac{s\sin\theta+\omega\cos\theta}{s^2+\omega^2}$
$\cos(\omega t+\theta)$	$\frac{s\cos\theta-\omega\sin\theta}{s^2+\omega^2}$
$e^{-\mathrm{at}}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{\mathrm{at}}\sin\omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
$e^{-\mathrm{at}}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$e^{\mathrm{at}}\cos\omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
sinhat	$\frac{a}{s^2-a^2}$
coshat	$\frac{s}{s^2-a^2}$
$\frac{d}{\mathrm{dt}}f(t)$	$\mathrm{sF}(s)-f(0-)$
$\frac{d^2}{\mathrm{dt}^2}f(t)$	$s^2F(s)-\mathrm{sf}(0-)-\frac{d}{\mathrm{dt}}f(0-)$
$\frac{d^3}{\mathrm{dt}^3}f(t)$	$s^3F(s)-s^2f(0-)-s\frac{d}{\mathrm{dt}}f(0-)-\frac{d^2}{\mathrm{dt}^2}f(0-)$

$\int f(t)\mathrm{d}t$	$\dfrac{F(s)}{s}$
$f(t-a)u(t-a)$	$e^{-\mathrm{as}}F(s)$
$e^{-\mathrm{at}}f(t)$	$F(s-a)$
$tf(t)$	$\dfrac{-d}{\mathrm{ds}}F(s)$
$t^n f(t)$	$(-1)^n \dfrac{d^n}{\mathrm{ds}^n}F(s)$

Transform of Periodic waveform $F(s) = \frac{1}{1-e^{-\mathrm{Ts}}} \times F_1(s)$

Initial Value Theorem $\lim\limits_{t\to 0} \mathrm{f}(t) = \lim\limits_{s\to\infty} [\mathrm{sF}(s)]$

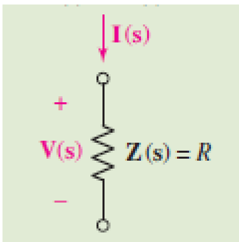
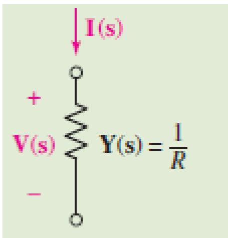
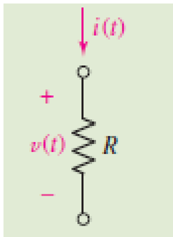
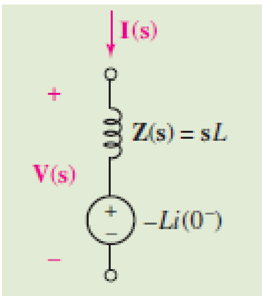
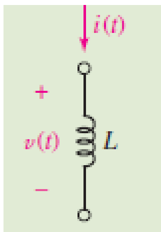
Final Value Theorem $\lim\limits_{t\to\infty} \mathrm{f}(t) = \lim\limits_{s\to 0} \mathrm{sF}(s)$

Convolution Integral $f(t) = L^{-1}[F_1(s)F_2(s)] = \int f_1(\tau)f_2(t-\tau) = \int f_2(\tau)f_1(t-\tau)$

Shifting Theorem

If $\mathrm{f}(t)\ \mathrm{u}(t) \leftrightarrow \mathrm{F}(s)$

Then f(t-a) u(t-a) $\leftrightarrow$ $\mathrm{e}^{\text{-as}}F(s)$

s-domain representation for a Resistor	$V(s) = RI(s)$  $I(s) = \frac{1}{R}V(s)$  	$v(t) = Ri(t)$ 
s-domain representation for an Inductor	$V(s) = sLI(s) - Li(0-)$  	$v(t) = L\frac{di}{dt}$ 

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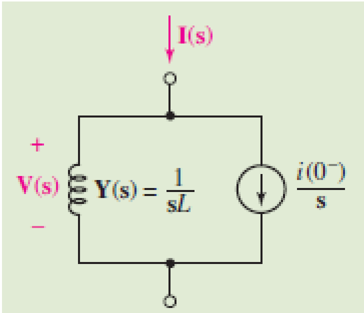
$$I(s) = \frac{V(s)}{sL} + \frac{i(0-)}{s}$$



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<td>s-domain representation for a Capacitor</td>
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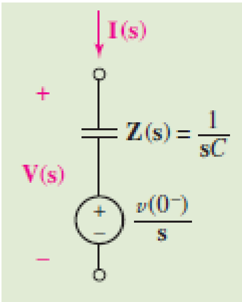
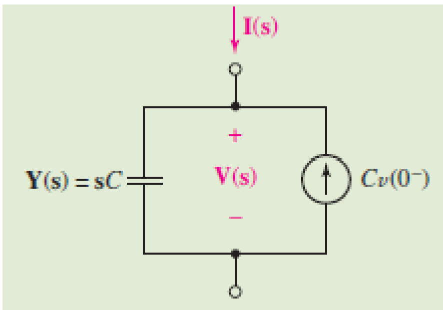
$$V(s) = \frac{I(s)}{sC} + \frac{v(0-)}{s}$$



$$I(s) = sCV(s) - Cv(0-)$$



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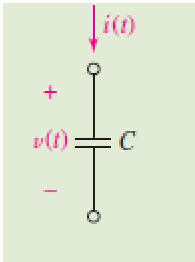
$$i(t) = C\frac{dV}{dt}$$



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Two Port Networks:

Network Parameters	Defining Equations
Z parameters	$V_1 = Z_{11}I_1 + Z_{12}I_2$ $V_2 = Z_{21}I_1 + Z_{22}I_2$
Y parameters	$I_1 = Y_{11}V_1 + Y_{12}V_2$ $I_2 = Y_{21}V_1 + Y_{22}V_2$
H parameters	$V_1 = h_{11}I_1 + h_{12}V_2$ $I_2 = h_{21}I_1 + h_{22}V_2$
T parameters (ABCD)	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$

Some parameter simplifications for passive, reciprocal Networks

Parameter	Condition for Reciprocal Networks	Condition for Symmetrical Network
z	$z_{12} = z_{21}$	$z_{11} = z_{22}$
y	$y_{12} = y_{21}$	$y_{11} = y_{22}$
ABCD	$\mathrm{AD} - \mathrm{BC} = 1$	
$A^1B^1C^1D^1$	$A^1D^1 - B^1C^1 = 1$	$A^1 = D^1$
h	$h_{12} = -h_{21}$	$\Delta_h = 1$
g	$g_{12} = -g_{21}$	$\Delta_g = 1$

Linear Wave shaping:

(i) Gain of High Pass RC Circuit $= \frac{1}{\sqrt{1+\left(\frac{f_{\mathrm{OL}}}{f}\right)^2}}$

(ii) Gain of Low pass RC Circuit $= \frac{1}{\sqrt{1+\left(\frac{f}{f_{\mathrm{OH}}}\right)^2}}$

(iii) General solution for a single time constant RC circuit : $v_o = v_f + (vi - v_f) \times e^{\frac{-t}{\tau}}$

(iv) Step Response to a high pass RC circuit. $v_o = \mathrm{V}e^{\frac{-t}{\mathrm{RC}}}$

(v) Step Response to a Low pass RC circuit:

$v_o = V(1 - e^{\frac{-t}{\mathrm{RC}}})$

Rise time $t_r = 2.2\mathrm{RC}$

(vi) Ramp input response to a high pass RC circuit $v_o = \alpha\mathrm{RC}(1 - e^{\frac{-t}{\mathrm{RC}}})$

(vii) Ramp input response to a low pass RC circuit:

$v_o = \alpha t - \alpha\mathrm{RC}\left(1 - e^{\frac{-t}{\mathrm{RC}}}\right)$