Signals and Systems (ECE – 2155, BME-2155)

1. Elementary signals

Name	Continuous time	Discrete time
Impulse function	$\delta(t) = 0, t \neq 0$	$\delta[n] = \begin{cases} 1, n = 0 \\ 0, n \neq 0 \end{cases}$
Unit step function	$\mathrm{u(t)} = \begin{cases} 1, \mathrm{t} \geq 0 \\ 0, \mathrm{t} < 0 \end{cases}$	$u[n] = \begin{cases} 1, n \geq 0 \\ 0, n < 0 \end{cases}$
Ramp	$r(\mathrm{t}) = \begin{cases} \mathrm{t}, \mathrm{t} \geq 0 \\ 0, \mathrm{t} < 0 \end{cases}$	$r[n] = \begin{cases} n, n \geq 0 \\ 0, n < 0 \end{cases}$
Exponential	$x(t) = e^{at}$	$x[n] = a^n$
Sinusoid	$x(t) = sin(\omega t + \phi)$	$x[n] = sin(\Omega n + \theta)$
Sinc function	$sinc(\omega_0 t) = \frac{sin(\pi\omega_0 t)}{\pi\omega_0 t}$	$sinc[\Omega_0 n] = \frac{sin(\pi\Omega_0 n)}{\pi\Omega_0 n}$
Rectangular pulse	$x(t) = \begin{cases} 1, \lvert t\rvert \leq T_0 \\ 0, \lvert t\rvert > T_0 \end{cases}$	$x[n] = \begin{cases} 1, \lvert n\rvert \leq M \\ 0, otherwise \end{cases}$
Triangular pulse	$\Lambda\left(\frac{t}{\tau}\right) = \begin{cases} 1 - \left\lvert\frac{t}{\tau}\right\rvert, t \leq \lvert\tau\rvert \\ 0, t > \lvert\tau\rvert \end{cases}$	$\Lambda\left[\frac{n}{N}\right] = \begin{cases} 1 - \frac{\lvert n\rvert}{N}, \lvert n\rvert \leq N \\ 0, otherwise \end{cases}$

2. Important properties of signals

Name	Properties	
	Continuous time	Discrete time
Impulse properties	$\int_{t=-\infty}^{\infty} \delta(t)dt = 1$ $\int_{t=-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0)$ $x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$ $\delta(at) = \frac{1}{\lvert a\rvert}\delta(t)$	$x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0]$

Even and Odd symmetry	$x_e(t) = x_e(-t)$ $x_o(t) = -x_o(-t)$ $x_e(t) = \frac{x(t) + x(-t)}{2}$ $x_o(t) = \frac{x(t) - x(-t)}{2}$	$x_e[n] = x_e[-n]$ $x_o[n] = -x_o[-n]$ $x_e[n] = \frac{x[n] + x[-n]}{2}$ $x_o[n] = \frac{x[n] - x[-n]}{2}$
Energy of non-periodic signals	$E = \int_{t=-\infty}^{\infty} \lvert x(t)\rvert^2 dt$	$E = \sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2$
Average Power of periodic signals	$P = \frac{1}{T} \int_{t=-T/2}^{T/2} \lvert x(t)\rvert^2\, dt$	$P = \frac{1}{N} \sum_{n=0}^{N-1} \lvert x[n]\rvert^2$
Linear combination of M signals	$\sum_{i=1}^{N} a_i x_i(t)$	$\sum_{i=1}^{N} a_i x_i[n]$
Convolution between two non-periodic signals	$x_1(t) * x_2(t) = \int_{\tau=-\infty}^{\infty} x_1(\tau) x_2(t-\tau) d\tau$	$x_1[n] * x_2[n] = \sum_{k=-\infty}^{\infty} x_1[k] x_2[n-k]$

3. LTI system analysis

	Continuous time system	**Discrete time system**
Name	$x(t) : input$ $y(t) : output$ $h(t) : impulse\ response$	$x[n] : input$ $y[n] : output$ $h[n] : impulse\ response$
Causality and stability in-terms of Impulse response	*Causality:* $h(t) = 0, t < 0$ $stability$: $\int_{t=-\infty}^{t} \lvert h(t)\rvert d\tau < \infty$ *memoryless:* $h(t) = c\delta(t)$	*Causality:* $h[n] = 0, n < 0$ $stability$: $\sum_{n=-\infty}^{n} \lvert h[n]\rvert < \infty$ *memoryless:* $h[n] = c\delta[n]$
Response to any input	$y(t) = x(t) * h(t)$	$y[n] = x[n] * h[n]$

Step response	$s(t) = \int_{\tau=-\infty}^{t} h(\tau) d\tau$	$s[n] = \sum_{m=-\infty}^{n} h[n]$
Differential/Difference equation description	$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x(t)\ ; a_0 \neq 0$	$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]\ ; a_0 \neq 0$

4. Geometric Series Formulas

1	$\sum_{n=0}^{M} a^n = \frac{1-a^{M+1}}{1-a}$
2	$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \qquad \lvert a < 1 \rvert$
3	$\sum_{n=M_1}^{M_2} a^n = \frac{a^{M_1} - a^{M_2+1}}{1-a}$
4	$\sum_{n=1}^{M} n = \frac{M(M+1)}{2}$

5. Fourier representation of signals

Time domain representation		Fourier representation		
$x(t) = \sum_{k=-\infty}^{\infty} X(k) e^{jk\omega_0 t}$	• Continuous • Periodic • Period=T, Fundamental frequency ω_0=2π/T rad/sec	$X(k) = \frac{1}{T} \int_{t=-T/2}^{T/2} x(t)\, e^{-jk\omega_0 t} dt$	• Discrete • Non periodic	FS
$x(t) = \frac{1}{2\pi} \int_{\omega=-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$	• Continuous • Non-Periodic	$X(j\omega) = \int_{t=-\infty}^{\infty} x(t) e^{-j\omega t} dt$	• Continuous • Non periodic	FT

$x[n] = \sum_{k=<N>} X(k) e^{jk\Omega_0 n}$	• Discrete • Periodic • Period=N, Fundamental frequency Ω_0=2π/N rad	$X(k) = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\Omega_0 n}$	• Discrete • Periodic • Period=N	DTFS
$x[n] = \frac{1}{2\pi} \int_{\Omega=-\pi}^{\pi} X(\Omega) e^{j\Omega n} d\Omega$	• Discrete • Non- Periodic	$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$	• Continuous • Periodic • Period= 2π rad	DTFT

6. Properties of Fourier Representations

6.1. Properties of Fourier Representation of Non-periodic signals

Property	*Continuous time signals*		*Discrete time signals*	
	Time domain	Frequency domain (FT)	Time domain	Frequency domain (DTFT)
Notation	$x(t)$ $x_i(t)$	$X(j\omega)$ $X_i(j\omega)$	$x[n]$ $x_i[n]$	$X(\Omega)$ $X_i(\Omega)$
Linearity	$\sum_{i=1}^{N} a_i x_i(t)$	$\sum_{i=1}^{N} a_i X_i(j\omega)$	$\sum_{i=1}^{N} a_i x_i[n]$	$\sum_{i=1}^{N} a_i X_i(\Omega)$
Time shifting	$x(t-t_0)$	$e^{-j\omega t_0} X(j\omega)$	$x[n-n_0]$	$e^{-j\Omega n_0} X(\Omega)$
Frequency shift	$e^{j\gamma t} x(t)$	$X(j(\omega-\gamma))$	$e^{j\Gamma n} x[n]$	$X(\Omega-\Gamma)$
Time reversal	$x(-t)$	$X(-j\omega)$	$x[-n]$	$X(-\Omega)$
Correlation	$r_{x_1 x_2}(\tau)$ $= x_1(\tau) * x_2(-\tau)$	$X_1(j\omega) X_2(-j\omega)$	$r_{x_1 x_2}[l]$ $= x_1[l] * x_2[-l]$	$X_1(\Omega) X_2(-\Omega)$
Differentiation in time	$\frac{d}{dt} x(t)$	$j\omega X(j\omega)$	--	--
Differentiation in frequency	$-jtx(t)$	$\frac{d}{d\omega} X(j\omega)$	$-jnx[n]$	$\frac{d}{d\Omega} X(\Omega)$
Integration /summation	$\int_{\tau=-\infty}^{t} x(\tau) d\tau$	$\frac{X(j\omega)}{j\omega} + \pi X(j0)\delta(\omega)$	$\sum_{m=-\infty}^{n} x[l]$	$\frac{X(\Omega)}{1-e^{j\Omega}} + \pi X(\Omega) \sum_{m=-\infty}^{\infty} \delta(\Omega - m2\pi)$
Convolution	$x_1(t) * x_2(t)$	$X_1(j\omega) X_2(j\omega)$	$x_1[n] * x_2[n]$	$X_1(\Omega) X_2(\Omega)$
Multiplication	$x_1(t) x_2(t)$	$\frac{1}{2\pi} \int_{\vartheta=-\infty}^{\infty} X_1(j\vartheta) X_2(j(\omega-\vartheta)) d\vartheta$	$x_1[n] x_2[n]$	$\frac{1}{2\pi} \int_{\lambda=-\pi}^{\pi} X_1(\lambda) X_2(\Omega-\lambda) d\lambda$

Symmetry	$x(t)\ real$	$X^*(j\omega) = X(-j\omega)$	$x[n]\ real$	$X^*(\Omega) = X(-\Omega)$
	$x(t)\ imaginary$	$X^*(j\omega) = -X(-j\omega)$	$x[n]\ imaginary$	$X^*(\Omega) = -X(-\Omega)$
	$x(t)\ real\ \&\ even$	$Im\{X(j\omega)\} = 0$	$x[n]\ real\ \&\ even$	$Im\{X(\Omega)\} = 0$
	$x(t)\ real\ \&\ odd$	$Re\{X(j\omega)\} = 0$	$x(n)\ real\ \&\ odd$	$Re\{X(\Omega)\} = 0$
Parseval's Theorem	$\int_{t=-\infty}^{\infty} \lvert x(t)\rvert^2 dt = \frac{1}{2\pi}\int_{\Omega=-\infty}^{\infty} \lvert X(j\omega)\rvert^2 d\Omega$		$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{\omega=-\pi}^{\pi} \lvert X(\Omega)\rvert^2 d\omega$	
Duality	$X(jt) \xleftrightarrow{FT} 2\pi x(-\omega)$		$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$ $X(e^{jt}) \xleftrightarrow{FS;1} x[-k]$	

6.2. Properties of Fourier Representation of periodic signals:

Continuous time signal: Periodic, Period=T, Fundamental frequency $\omega_0 = \frac{2\pi}{T}\ radian/sec$

Discrete time signal: Periodic, Period=N, Fundamental frequency $\Omega_0 = \frac{2\pi}{N}\ radian$

Property	*Continuous time signals*		*Discrete time signals*	
	Time domain	Frequency domain (FS)	Time domain	Frequency domain (DTFS)
Notation	$x(t)$ $x_i(t)$	$X(k)$ $X_i(k)$	$x[n]$ $x_i[n]$	$X(k)$ $X_i(k)$
Linearity	$\sum_{i=1}^{N} a_i x_i(t)$	$\sum_{i=1}^{N} a_i X_i(k)$	$\sum_{i=1}^{N} a_i x_i[n]$	$\sum_{i=1}^{N} a_i X_i(k)$
Time shifting	$x(t-t_0)$	$e^{-jk\omega_0 t_0}X(k)$	$x[n-n_0]$	$e^{-jk\Omega_0 n_0}X(k)$
Frequency shift	$e^{jk_0\omega_0 t}x(t)$	$X(k-k_0)$	$e^{jk_0\Omega_0 n}x[n]$	$X(k-k_0)$
Differentiation in time	$\frac{d}{dt}x(t)$	$jk\omega_0 X(k)$	--	--
Convolution	$x_1(t) * x_2(t)$	$TX_1(k)X_2(k)$	$x_1[n] * x_2[n]$	$NX_1(k)X_2(k)$
Multiplication	$x_1(t)x_2(t)$	$\sum_{l=-\infty}^{\infty} X_1(l)X_2(k-l)$	$x_1[n]x_2[n]$	$\sum_{l=0}^{N-1} X_1(l)X_2(k-l)$
Symmetry	$x(t)\ real$	$X^*(k) = X(-k)$	$x[n]\ real$	$X^*(k) = X(-k)$
	$x(t)\ imaginary$	$X^*(k) = -X(-k)$	$x[n]\ imaginary$	$X^*(k) = -X(-k)$
	$x(t)\ real\ \&\ even$	$Im\{X(k)\} = 0$	$x[n]\ real\ \&\ even$	$Im\{X(k)\} = 0$
	$x(t)\ real\ \&\ odd$	$Re\{X(k)\} = 0$	$x[n]\ real\ \&\ odd$	$Re\{X(k)\} = 0$
Parseval's Theorem	$\frac{1}{T}\int_{t=0}^{T} \lvert x(t)\rvert^2 dt = \sum_{k=-\infty}^{\infty} \lvert X(k)\rvert^2$		$\frac{1}{N}\sum_{n=0}^{N-1} \lvert x[n]\rvert^2 = \sum_{k=0}^{N-1} \lvert X(k)\rvert^2$	
Duality	$x[n] \xleftrightarrow{DTFT} X(e^{j\omega})$ $X(e^{jt}) \xleftrightarrow{FS;1} x[-k]$		$X[n] \xleftrightarrow{DTFS;\ \omega_0} \frac{1}{N}x[-k]$	

6.3. Relating different Fourier representations:

Let

$$g(t) \xleftrightarrow{FS;\ \omega_0=\frac{2\pi}{T}} G[k]$$
$$v[n] \xleftrightarrow{DTFT} V(\omega)$$
$$w[n] \xleftrightarrow{DTFS;\ \Omega_0=\frac{2\pi}{N}} W[k]$$

6.3.1. FT representation for a continuous-time periodic signal

$$g(t) \xleftrightarrow{FT} G(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} G[k]\,\delta(\omega - k\omega_0)$$

6.3.2. DTFT representation for a discrete-time periodic signal

$$w[n] \xleftrightarrow{DTFT} W(\Omega) = 2\pi \sum_{k=-\infty}^{\infty} W[k]\,\delta(\Omega - k\Omega_0)$$

6.3.3 FT representation for a discrete-time periodic signal

$$\sum_{n=-\infty}^{\infty} v[n]\,\delta(t - nT_s) \xleftrightarrow{FT} V_\delta(j\omega) = \sum_{n=-\infty}^{\infty} v[n]e^{-j\omega nT_s}$$
$$(T_s\ is\ a\ fixed\ number)$$

7. **Sampling:**

Continuous time signal x(t) with FT $X(j\omega)$ is sampled at sampling interval T_s to get $x_\delta(t)$

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x[nT_s]\,\delta(t - nT_s) \xleftrightarrow{FT} X_\delta(j\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X\left(j\left(\omega - k\frac{2\pi}{T_s}\right)\right)$$

8. Basic DTFS pairs

Time domain	Frequency domain
$x(n) = \sum_{k=<N>} X(k) e^{jk\Omega_0 n}$ $Period = N$	$X(k) = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\Omega_0 n}$ $\Omega_0 = \frac{2\pi}{N}$
$x[n] = \begin{cases} 1, \lvert n\rvert \leq M \\ 0, \quad M < \lvert n\rvert \leq \frac{N}{2} \end{cases}$ $x[n] = x[n+N]$	$X(k) = \frac{sin\left(k\frac{\Omega_0}{2}(2M+1)\right)}{N\, sin\left(k\frac{\Omega_0}{2}\right)}$
$x[n] = e^{jp\Omega_0 n}$	$X[k] = \begin{cases} 1, k = p, p \pm N, p \pm 2N \ldots\ldots \\ 0, otherwise \end{cases}$
$x[n] = cos(p\Omega_0 n)$	$X(k) = \begin{cases} \frac{1}{2}, k = \pm p, \pm p \pm N, \pm p \pm 2N \ldots\ldots \\ 0, otherwise \end{cases}$
$x[n] = sin(p\Omega_0 n)$	$X(k) = \begin{cases} \frac{1}{2j}, k = p, p \pm N, p \pm 2N \ldots\ldots \\ \frac{-1}{2j}, k = -p, -p \pm N, -p \pm 2N \ldots\ldots \\ 0, otherwise \end{cases}$
$x[n] = 1$	$X[k] = \begin{cases} 1, k = 0, \pm N, \pm 2N \ldots\ldots \\ 0, otherwise \end{cases}$
$x[n] = \sum_{p=-\infty}^{\infty} \delta[n - pN]$	$X(k) = \frac{1}{N}$

9. Basic FS pairs

Time domain	Frequency domain
$x(t) = \sum_{k=-\infty}^{\infty} X(k) e^{jk\omega_0 t}$ $Period = T$	$X(k) = \frac{1}{T} \int_{t=-\frac{T}{2}}^{\frac{T}{2}} x(t)\, e^{-jk\omega_0 t} dt$ $\omega_0 = \frac{2\pi}{T}$
$x(t) = \begin{cases} 1, \lvert t\rvert \leq T_0 \\ 0, T_0 < \lvert t\rvert \leq \frac{T}{2} \end{cases}$	$X(k) = \frac{sin(k\omega_0 T_0)}{k\pi}$
$x(t) = e^{jp\omega_0 t}$	$X(k) = \delta[k-p]$
$x(t) = cos(p\omega_0 t)$	$X(k) = \frac{1}{2}\delta[k-p] + \frac{1}{2}\delta[k+p]$
$x(t) = sin(p\omega_0 t)$	$X(k) = \frac{1}{2j}\delta[k-p] - \frac{1}{2j}\delta[k+p]$
$x(t) = \sum_{p=-\infty}^{\infty} \delta(t - pT)$	$X(k) = \frac{1}{T}$

10. Basic DTFT pairs

Time domain	Frequency domain
$x(n) = \frac{1}{2\pi}\int_{\Omega=-\pi}^{\pi} X(\Omega)e^{j\Omega n}d\Omega$	$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$
$x[n] = \begin{cases}1, \lvert n\rvert \le M \\ 0, otherwise\end{cases}$	$X(\Omega) = \frac{sin\left[\Omega\left(\frac{2M+1}{2}\right)\right]}{sin\left(\frac{\Omega}{2}\right)}$
$x[n] = a^n u[n], \quad \lvert a\rvert < 1$	$X(\Omega) = \frac{1}{1-ae^{-j\Omega}}$
$x[n] = \delta[n]$	$X(\Omega) = 1$
$x[n] = u[n]$	$X(\Omega) = \frac{1}{1-e^{-j\Omega}} + \pi\sum_{p=-\infty}^{\infty}\delta(\Omega - 2\pi p)$
$x[n] = \frac{1}{\pi n}sin(Wn), \quad 0 < W < \pi$	$X(\Omega) = \begin{cases}1, \lvert\Omega\rvert \le W \\ 0, W < \lvert\Omega\rvert \le \pi\end{cases}$ $X(\Omega)$ *is* 2π *periodic*
$x[n] = (n+1)a^n u[n]$	$X(\Omega) = \frac{1}{(1-ae^{-j\Omega})^2}$
$x[n] = cos(\Omega_0 n)$	$X(\Omega) = \pi\sum_{k=-\infty}^{\infty}\delta(\Omega - \Omega_0 - k2\pi) + \delta(\Omega + \Omega_0 - k2\pi)$
$x[n] = sin(\Omega_0 n)$	$X(\Omega) = \frac{\pi}{j}\sum_{k=-\infty}^{\infty}\delta(\Omega - \Omega_0 - k2\pi) - \delta(\Omega + \Omega_0 - k2\pi)$
$x[n] = e^{\Omega_0 n}$	$X(\Omega) = 2\pi\sum_{k=-\infty}^{\infty}\delta(\Omega - \Omega_0 - k2\pi)$
$x[n] = \sum_{k=-\infty}^{\infty}\delta(n - kN)$	$X(\Omega) = \frac{2\pi}{N}\sum_{k=-\infty}^{\infty}\delta\left(\Omega - k\frac{2\pi}{N}\right)$

11. Basic FT pairs

Time domain	Frequency domain
$x(t) = \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$	$X(j\omega) = \int_{t=-\infty}^{\infty} x(t)e^{-j\omega t}dt$
$x(t) = \begin{cases}1, \lvert t\rvert \le T_0 \\ 0, \lvert t\rvert > T_0\end{cases}$	$X(j\omega) = \frac{2sin(\omega T_0)}{\omega}$
$x(t) = \frac{1}{\pi t}sin(Wt)$	$X(j\omega) = \begin{cases}1, \lvert\omega\rvert \le W \\ 0, \text{otherwise}\end{cases}$
$x(t) = \delta(t)$	$X(j\omega) = 1$
$x(t) = 1$	$X(j\omega) = 2\pi\delta(\omega)$
$x(t) = u(t)$	$X(j\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$
$x(t) = e^{-at}u(t) \quad Re\{a\} > 0$	$X(j\omega) = \frac{1}{a + j\omega}$

$x(t) = te^{-at}u(t), \quad Re\{a\} > 0$	$X(j\omega) = \frac{1}{(a+j\omega)^2}$
$x(t) = e^{-a\|t\|}, \quad a > 0$	$X(j\omega) = \frac{2a}{a^2+\omega^2}$
$x(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$	$X(j\omega) = e^{-\omega^2/2}$
$x(t) = cos(\omega_0 t)$	$X(j\omega) = \pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)$
$x(t) = sin(\omega_0 t)$	$X(j\omega) = \frac{\pi}{j}\delta(\omega-\omega_0) - \frac{\pi}{j}\delta(\omega+\omega_0)$
$x(t) = e^{j\omega_0 t}$	$X(j\omega) = 2\pi\delta(\omega-\omega_0)$
$x(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$	$X(j\omega) = \frac{2\pi}{T_s}\sum_{k=-\infty}^{\infty} \delta\left(\omega - k\frac{2\pi}{T_s}\right)$

12. Some common Laplace transform pairs:

Signal $x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}ds$	*Transform* $X(s) = \int_{t=-\infty}^{\infty} x(t)e^{-st}dt$	ROC
$u(t)$	$\frac{1}{s}$	$Re\{s\} > 0$
$-u(-t)$	$\frac{1}{s}$	$Re\{s\} < 0$
$tu(t)$	$\frac{1}{s^2}$	$Re\{s\} > 0$
$-tu(-t)$	$\frac{1}{s^2}$	$Re\{s\} < 0$
$\delta(t-\tau)$	$e^{-s\tau}$	*for all s*
$e^{-at}u(t)$	$\frac{1}{s+a}$	$Re\{s\} > -a$
$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$Re\{s\} < -a$
$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$	$Re\{s\} > -a$
$-te^{-at}u(-t)$	$\frac{1}{(s+a)^2}$	$Re\{s\} < -a$
$cos(\omega_0 t)u(t)$	$\frac{s}{s^2+\omega_0{}^2}$	$Re\{s\} > 0$
$sin(\omega_0 t)u(t)$	$\frac{\omega_0}{s^2+\omega_0{}^2}$	$Re\{s\} > 0$
$e^{-at}cos(\omega_0 t)u(t)$	$\frac{s+a}{(s+a)^2+\omega_0{}^2}$	$Re\{s\} > -a$
$e^{-at}sin(\omega_0 t)u(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0{}^2}$	$Re\{s\} > -a$

13. Laplace Transform properties:

Signal	*Unilateral Transform* $x(t) \xleftrightarrow{\mathcal{L}_u} X(s)$ $y(t) \xleftrightarrow{\mathcal{L}_u} Y(s)$	*Bilateral Transform* $x(t) \xleftrightarrow{\mathcal{L}} X(s)$ $y(t) \xleftrightarrow{\mathcal{L}} Y(s)$	*ROC* $s \in R_x$ $s \in R_y$
$ax(t) + by(t)$	$aX(s) + bY(s)$	$aX(s) + bY(s)$	$atleast\ R_x \cap R_y$
$x(t-\tau)$	$e^{-s\tau}X(s)$ $if\ x(t-\tau)u(t) = x(t-\tau)u(t-\tau)$	$e^{-s\tau}X(s)$	R_x
$e^{s_0}x(t)$	$X(s-s_0)$	$X(s-s_0)$	$R_x + Re\{s_0\}$
$x(at)$	$\frac{1}{a}X\left(\frac{s}{a}\right),\ \ a > 0$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	$\frac{R_x}{\lvert a\rvert}$
$x(t) * y(t)$	$X(s)Y(s)$ $if\ x(t) = y(t) = 0\ \ \ for\ t < 0$	$X(s)Y(s)$	$atleast\ R_x \cap R_y$
$-tx(t)$	$\frac{d}{ds}X(s)$	$\frac{d}{ds}X(s)$	R_x
$\frac{d}{dt}x(t)$	$sX(s) - x(0^-)$	$sX(s)$	$atleast\ R_x$
$\int_{\tau=-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}\int_{\tau=-\infty}^{0^-} x(\tau)d\tau + \frac{X(s)}{s}$	$\frac{X(s)}{s}$	$atleast\ R_x \cap \{Re\{s\} > 0\}$

14. Some common z-Transform pairs

Signal x[n]	*Transform* X(z)	
$x[n] = \frac{1}{2\pi j}\oint X(z)z^{n-1}dz$	$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$	ROC
$\delta[n]$	1	*All z*
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$nu[n]$	$\frac{z^{-1}}{(1-z^{-1})^2}$	$\lvert z\rvert > 1$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$

$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$cos[\Omega_0 n]u[n]$	$\frac{1-z^{-1}cos(\Omega_0)}{1-2z^{-1}cos(\Omega_0)+z^{-2}}$	$\lvert z\rvert > 1$
$sin[\Omega_0 n]u[n]$	$\frac{z^{-1}sin(\Omega_0)}{1-2z^{-1}cos(\Omega_0)+z^{-2}}$	$\lvert z\rvert > 1$
$a^n cos[\Omega_0 n]u[n]$	$\frac{1-az^{-1}cos(\Omega_0)}{1-2az^{-1}cos(\Omega_0)+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
$a^n sin[\Omega_0 n]u[n]$	$\frac{az^{-1}sin(\Omega_0)}{1-2az^{-1}cos(\Omega_0)+a\quad z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$

15. z-Transform Properties:

Property	Time Domain	z-Domain	ROC
Notation	$x[n]$ $x_i[n]$	$X(z)$ $X_i(z)$	ROC: $r_2 < \lvert z\rvert < r_1$ ROC$_i$
Linearity	$\sum_{i=1}^{N} a_i x_i[n]$	$\sum_{i=1}^{N} a_i X_i[z]$	At least intersection of ROC$_i$
Time shifting	$x[n-k]$	$z^{-k}X(z)$	That of $X(z)$, except Z = 0 if k>0 and z = ∞ if k<0
Scaling in z-domain	$a^n x[n]$	$X\left(\frac{z}{a}\right)$	$\lvert a\rvert r_2 < \lvert z\rvert < \lvert a\rvert r_1$
Time reversal	$x[-n]$	$X(z^{-1})$	$\frac{1}{r_1} < \lvert z\rvert < \frac{1}{r_2}$
Conjugation	$x^*[n]$	$X^*(z^*)$	ROC
Real part	$Re\{x[n]\}$	$\frac{1}{2}[X(z)+X^*(z^*)]$	Includes ROC
Imaginary part	$Im\{x[n]\}$	$\frac{1}{2}[X(z)-X^*(z^*)]$	Includes ROC
Differentiation in z-domain	$nx[n]$	$-z\frac{d}{dz}X(z)$	$r_2 < \lvert z\rvert < r_1$
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	$atleast\ ROC_1 \cap ROC_2$
Multiplication	$x_1[n]x_2[n]$	$\frac{1}{2\pi j}\oint X_1(\vartheta)X_2\left(\frac{z}{\vartheta}\right)\vartheta^{-1}d\vartheta$	$atleast\ r_{1l}r_{2l} < \lvert z\rvert < r_{1u}r_{2u}$
Initial value theorem	$If x[n] is\ causal$	$x[0] = \lim_{z\to\infty} X(z)$	
Parseval's relation	$\sum_{n=-\infty}^{\infty} x_1[n]x_2{}^*[n] = \frac{1}{2\pi j}\oint X_1(\vartheta)X_2{}^*\left(\frac{1}{\vartheta^*}\right)\vartheta^{-1}d\vartheta$		