Time: 3 Hours   **Date: 17 NOV 2021**   **Max. Marks: 50**

✓ **Answer ALL questions.**
✓ **Missing data, if any, may be suitably assumed.**

1 a. Solve $2x^2 y \dfrac{dy}{dx} = \tan(x^2 y^2) - 2xy^2$ *3M*

   b. Solve $x\sin x \dfrac{dy}{dx} + y(x\cos x - \sin x) = 2$ *3M*

   c. Solve $2xy'' + 3y' - \dfrac{y}{x} = 5 - \dfrac{1}{2}\dfrac{\sin(\log x)}{x}$ *4M*

2 a. Solve $u_{xx} + u_{xy} - 2u_{yy} = 0$ by Method of indicated transforms $v = x + y$, $z = 2x - y$.   *3M*

   b. Solve by R-K method of fourth order $\dfrac{dy}{dx} = \dfrac{y^2 - x^2}{y^2 + x^2}$ $y(0) = 1$ $at\ x = 0.2$ *3M*

   c. Solve by the method of variation of parameters $y'' + y = \sec^2 x$.   *4M*

3 a. Find the Laplace Transform of half wave rectified sine wave defined as   *3M*

$$f(t) = \begin{cases} \sin\omega t & 0 < t < \dfrac{\pi}{\omega} \\ 0 & \dfrac{\pi}{\omega} < t < \dfrac{2\pi}{\omega} \end{cases} \qquad f\left(t + \dfrac{2\pi}{\omega}\right) = f(t) \text{ Draw the graph.}$$

   b. Find $L^{-1}\left\{\dfrac{1}{(s-1)(s^2+1)}\right\}$ *3M*

   c.   *4M*

Solve the linear differential Equation $y'' + y = t^2 + 2t$, $y(0) = 4$, $y'(0) = -2$

4 a. Let $f(z) = u + iv$ be an analytic function Then show that   3M

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)^2| = 4|f'(z)|^2$$

  b. Evaluate for the Residues of $\oint_c \frac{dz}{z^2 e^{2z}}$ where $C:|z|=1$ 3M

  c. Find all the Taylor's and Laurent's series of $f(z) = \frac{3-2z}{Z^2 - 3z + 2}$ at $z = 0$ 4M

5 a. Solve $(D^3 - 2D^2 - 5D + 6)y = 0$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = 1$.   3M

  b. State and prove Cauchy's integral formula.   3M

  c. Find $L\left\{ \frac{1 - \cos at}{t} \right\}$ 4M