MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent institution of MAHE, Manipal)

SEMESTER-I, M. TECH (DEFENCE TECHNOLOGY)
END-SEMESTER EXAMINATION, FEB./MARCH, 2022
COURSE: Advanced Engineering Mathematics (MAT5164)

Duration: 3 Hrs **Date: 25/02/2022** **MAX. MARKS: 50**

Note:

- *All questions are compulsory*
- *Draw a neat diagram wherever necessary*
- *Stepwise answers carry marks*

Q1a. Define Boundary Value Problem. Amongst all the numerical methods for solving ODE, which one is the most accurate? **[2M]**

Q1b. (i) Mention the factors to decide the type of attack on a target. **[3M]**
(ii) What policy India should adopt against the repeated terrorist attacks from Pakistan?

Q1c. Given **[5M]**
$\frac{dy}{dx} - \sqrt{xy} = 2, \; y(1) = 1;$
Find the value of $y(2)$ in steps of 0.1 using Euler's modified method.

Q2a. If u is a function of x and y, write the central difference approximation formula for $\frac{\partial u}{\partial x}$. **[2M]**

Q2b. If $P_n(x)$ is a Legendre polynomial of degree n and α is such that $P_n(\alpha) = 0$. Show that $P_{n-1}(\alpha)$ and $P_{n+1}(\alpha)$ are of opposite signs. **[3M]**

Q2c. Use the Bender-Schmidt method to solve the following PDE: **[5M]**

$$u_t = 5u_{xx}$$

Given

$$u(0,t) = 0, \qquad u(5,t) = 60,$$

and

$$u(x,0) = \begin{cases} 20x, & 0 \le x \le 3 \\ 60, & 3 \le x \le 5 \end{cases}$$

With $h = 1$ and $l = 0.5$.
Find the value of $u(x,t)$ for $0 \le x \le 3$ and $0 \le t \le 1$.

Q3a. Define half range Fourier series. **[2M]**

Q3b. The average number of accidents at a level crossing every year is 5. Calculate the probability that there are exactly 3 accidents there this year. **[3M]**

Q3c. A company has installed 10,000 electric bulbs in a metro. These bulbs have an average life of 1000 burning hours with a standard deviation 200 hours. Assuming normality what number of bulbs might fail **[5M]**

(i) In the first 800 burning hours?
(ii) Between 800 and 1200 burning hours?

After what period of burning hours would you expect that

(iii) 10% of the bulbs would fail?
(iv) 10% of the bulbs would survive?

Q4a. Write few lines about a topic which you liked most in this subject. **[2M]**

Q4b. Find the largest eigen value and corresponding eigen vector of the following matrix: **[3M]**

$$A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix},$$

Assuming the initial eigen vector $X^{(0)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$.

Q4c. Find the general solution of the given system of ODE by using eigen value eigen vector method: **[5M]**

$$x_1' = 3x_1 - x_3$$
$$x_2' = -2x_1 + 2x_2 + x_3$$
$$x_3' = 8x_1 - 3x_3$$

Q5a. Define Reliability and express it mathematically. **[2M]**

Q5b. **[3M]**

(i) Write down the differential equations for the modern counter-insurgency (COIN) model.
(ii) Derive the phase trajectories for (i) $\mu = 1$; (ii) $\mu = 0$.
(iii) What happens in the case (ii), when R tends to zero, that is, R force is nearly wiped out?

Q5c. Define simulation and list four advantages of simulation. **[5M]**