Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

FIRST SEMESTER M.TECH (SOFTWARE ENGINEERING) DEGREE END SEMESTER EXAMINATION-FEB 2022
SUBJECT:MATHEMATICAL LOGIC (ICT 5153)
(REVISED CREDIT SYSTEM)

TIME: 1.15 HOURS **09/02/2022** **MAX. MARKS: 20**

Instructions to candidates
- Answer **ALL** questions.
- Missing data, if any may be suitably assumed.

PART-B

1A. Consider the argument "*If it has snowed, it will be poor driving. If it is poor driving, I will be late unless I start early. Indeed, it has snowed. Therefore, I must start early to avoid being late*". Use the following atoms:

s : it has snowed

p : it is poor driving

l : I will be late, and

e : I start early

to write the given argument as the sequent in propositional logic. Prove the sequent using natural deduction rules. [5]

1B. Prove the equivalence $\forall x\phi \wedge \forall x\psi \dashv\vdash \forall x(\phi \wedge \psi)$. [3]

1C. State which of the following strings are well-formed CTL formulas. For those which are well-formed, draw the parse tree. For those which are not well-formed, explain why not.

i) $\neg(\neg p) \vee (r \wedge s)$
ii) Xq
iii) $\neg AXq$
iv) $p\, U(AX\bot)$

[2]

2A. Using the natural deduction rules for $KT45^n$, prove the validity of

i) $K_i(p \wedge q) \leftrightarrow K_i p \wedge K_i q$
ii) $C(p \wedge q) \leftrightarrow Cp \wedge Cq$

[5]

2B. Consider the Kripke model $\mathcal{M}$ of Figure Q.2B. Indicate for each of the following LTL formulae the set of states for which these formulae hold the relation $\mathcal{M} \vDash \phi$.

i) Xa
ii) $XXXa$
iii) Gb
iv) GFa
v) $G(b\ U\ a)$
vi) $F(a\ U\ b)$

2C. Let $\mathcal{F} \stackrel{def}{=} \{i\}$ and $\mathcal{P} \stackrel{def}{=} \{R, F\}$; where i is a constant, F a predicate symbol with arity one and R a pedicate symbol with arity two. A model $\mathcal{M}$ contains a set of concrete elements A-which may be a set of states of a computer program. The interpretations $i^{\mathcal{M}}$, $R^{\mathcal{M}}$, and $F^{\mathcal{M}}$ may then be a designated initial state, a state transition relation, and a set of final (accepting) states, respectively. Let $A \stackrel{def}{=} \{a, b, c\}$, $i^{\mathcal{M}} \stackrel{def}{=} a$, $R^{\mathcal{M}} \stackrel{def}{=} \{(a, a), (a, b), (a, c), (b, c), (c, c)\}$, and $F^{\mathcal{M}} \stackrel{def}{=} \{b, c\}$. For the given model $\mathcal{M}$, check the satisfaction relation $\mathcal{M} \vDash \phi$, where ϕ :

i) $\forall x \forall y \forall z (R(x, y) \wedge R(x, z) \rightarrow y = z)$

ii) $\forall x \exists y R(x, y)$

[2]

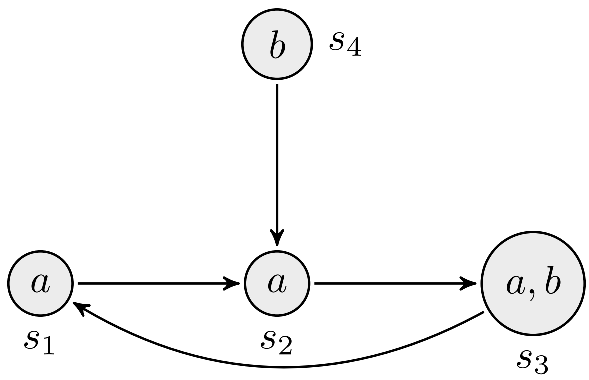

Figure: Q.2B