INTERNATIONAL CENTRE FOR APPLIED SCIENCES
MAHE, MANIPAL
B.Sc. (Applied Sciences) in Engg.
End – Semester Theory Examinations – NOV 2021
I SEMESTER - MATHEMATCS-I (IMA 111)

Time: 3 Hours **Date: 18 NOV 2021** **Max. Marks: 50**

- ✓ **Answer ALL questions.**
- ✓ **Missing data, if any, may be suitably assumed.**

1A. For the cardiode $r = a(1 + cos\theta)$, show that $\frac{\rho^2}{r} = constant$. Deduce that if ρ_1 and ρ_2 are radii of curvature at the extremities of a chord through the pole, then ${\rho_1}^2 + {\rho_2}^2 = \frac{16a^2}{9}$.

1B. Find the evolute of astroid $x = acos^3\theta$ and $y = asin^3\theta$.

1C. Prove that for any quadratic function $px^2 + qx + r$, the value of θ in lagranges mean value theorem is always half for every interval $[a, b]$.

4+3+3

2A. If $y^{1/m} + y^{-1/m} = 2x$. Prove that

$(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0.$

2B. Expand $y = \log_e(1 + sin^2x)$ in macluarin's series upto term containing x^6.

2C. Evaluate $\int_0^\infty \frac{x^2}{(1+x^2)^{\frac{7}{2}}} dx$

4+3+3

3A. Consider the following census data

Year:	1961	1971	1981	1991	2001
Population in 1000's:	22	26	32	44	48

Estimate the change in population from 1998 – 2000.

3B. Find the angle of intersection of the curves $r = alog\theta$ and $r = \frac{a}{log\theta}$

3C. Test for convergence or divergence of the following series

$$\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \left(\frac{5^5}{4^5} - \frac{5}{4}\right)^{-4} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

4+3+3

4A. Trace the curve $xy^2 = a^2(a - x)$

4B. Using Newtons divided difference formula evaluate f(8) and f(15)

x:	4	5	7	10	11	13
f(x):	48	100	294	900	1210	2028

4C. Evaluate $\lim_{x\to 0} \frac{[1+x]^{1/x} - e^{-\frac{ex}{2}}}{x^2}$.

4+3+3

5A. Find the interval of convergence of the following series

$$x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \cdots \ldots \ldots$$

5B. Find the area of the region lying above x-axis and included between the circle $x^2 + y^2 = 2ax$ and the parabola $y^2 = ax$.

5C. Find the perimeter of the curve $r = a(cos\theta + \sin\theta),\ 0 \le \theta \le \pi$.

4+3+3

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