Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
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THIRD SEMESTER B.TECH. (AUTOMOBILE ENGINEERING)
MAKEUP EXAMINATIONS JAN 2022

SUBJECT: ENGINEERING MATHEMATICS IV [MAT 2151]

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** the questions.
- Missing data may be suitably assumed.

1A.	Solve $x^2y'' + xy' + (x^2-3)y = 0,\ \ y(1) = 0,\ y(2) = 2$ with $h = 0.25$.	**4M**
1B.	With step size $h = 0.5$, solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -1,\ |x| < 1,\ |y| < 1,$ $u(1,y) = 0,\ u(-1,y) = 0,\ u(x,1) = 0, u(x,-1) = 0.$	**3M**
1C.	Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2},\ 0 < x < 1,\ t > 0$ with $u(x,\ 0) = \frac{\partial u}{\partial x}(x,0) = 0,\ u(0,t) = 0,\ u(1,t) = 100\sin \pi t$. Compute u for four-time step with $h = 0.25$.	**3M**
2A.	With $h = 0.25$, solve following equation using Crank-Nicolson's method $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}; 0 < x < 1; t > 0; u(x,0) = 100(x - x^2); u(0,t) = 0 = u(1,t)$ for one time steps.	**4M**
2B.	Obtain Fourier expansion of $f(x) = \frac{1}{2}(\pi - x)$ in $-\pi < x < \pi$.	**3M**
2C.	Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following data by Harmonic analysis. (x values are in terms of degree) x: 0, 45, 90, 135, 180, 225, 270, 315 y: 2, 3/2, 1, 1/2, 0, 1/2, 1, 3/2	**3M**
3A.	Find the Fourier Transform of $e^{-a^2x^2},\ a > 0$ and Hence deduce that $F\left(e^{-\frac{x^2}{2}}\right) = e^{-\frac{s^2}{2}}$.	**4M**
3B.	Find the directional derivative of the equation $\phi = x^2yz + 4xz^2$ at $(1,-2,-1)$ along $2i - j - 2k$.	**3M**
3C.	Find the value of the constant 'a' such that	**3M**

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	$\vec{F} = (axy - z^3)i + (a-2)x^2 j + (1-a)xz^2 k$ is irrotational and hence find a scalar function such that $\nabla\emptyset = \vec{F}$.	
4A.	Apply Green's theorem to evaluate $\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$, where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$.	**4M**
4B.	If a force $\vec{F} = 2x^2 y\,\hat{\imath} + 3xy\hat{\jmath}$ displaces a particle in the XY-plane from (0, 0) to (1, 4) along a curve $y = 4x^2$. Find the work done.	**3M**
4C.	Evaluate $\iint_S F.n\,dS$ where $F = 4xzi - y^2 j + yzk$ where S is the surface of the cube bounded by $x = 0,\ x = 1, y = 0,\ y = 1,\ z = 0,\ z = 1$.	**3M**
5A.	Verify Stoke's theorem for $F = (2x - y)i + yz^2 j - y^2 zk$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary	**4M**
5B.	Solve $y^3 \frac{\partial z}{\partial x} + x^2 \frac{\partial z}{\partial y} = 0$, by method of separation variables.	**3M**
5C.	Solve $u_{xx} + u_{xy} - 2u_{yy} = 0$ using the transformation $v = x + y$, $z = 2x - y$.	**3M**