ECE 3154 Microwave Engineering

Microwave Components

Parameters	Formulas
For Directional coupler, Coupling Factor (dB) Directivity (dB)	$10\log_{10}\frac{P_1}{P_4}$ $10\log_{10}\frac{P_4}{P_3}$ P_1 is power input to port 1 P_3 is power output from port 3 P_4 is power output from port 4
Spacing between the centres of two holes of a two hole directional coupler is	$L = (2n+1)\frac{\lambda_g}{4}$ Where n is any positive integer.
Efficiency of klystron	$\frac{\beta_0 I_2 V_2}{2 I_0 V_0}$ Where β_0 is the beam coupling coefficient of the catcher gap V_2 is the fundamental component of the catcher gap voltage I_0 is the DC current V_0 is the high dc voltage
Depth of velocity modulation in klystron	$\frac{\beta_i V_1}{V_0}$ β_i is the beam coupling coefficient V_1 is the amplitude of the signal.
Output power delivered to the catcher cavity and the load for a klystron is	$\frac{\beta_0 I_2 V_2}{2}$

Fundamental Parameters of Antenna

Parameter	Formula
Infinitesimal area of sphere dA	$dA = r^2 \sin\theta \, d\theta \, d\phi \qquad (m^2)$ Where r is radius of sphere
Elemental solid angle of sphere $d\Omega$	$d\Omega = \sin\theta \, d\theta \, d\phi \qquad (sr)$
Average power density $\mathbf{W}_{av}$	$\mathbf{W}_{av} = \frac{1}{2}\,\mathrm{Re}[\mathbf{E} \times \mathbf{H}^*] \qquad (W/m^2)$
Radiated power/average radiated power P_{rad}	$P_{rad} = P_{av} = \oiint_s \mathbf{W}_{av} \cdot d\boldsymbol{s} = \frac{1}{2}\oiint_s \mathrm{Re}[\mathbf{E} \times \mathbf{H}^*] \cdot d\boldsymbol{s}$
Approximate maximum directivity (*omnidirectional pattern*)	$D_0 \simeq \frac{101}{HPBW\ (degrees) - 0.0027[HPBW\ (degrees)]^2}$ (McDonald)

	$$D_0 \simeq -172.4 + 191 \sqrt{0.818 + \frac{1}{HPBW\ (degrees)}}$$ (Pozar)
Loss resistance R_L (*straight wire/uniform current*)	$$R_L = R_{hf} = \frac{l}{P} R_s = \frac{l}{P}\sqrt{\frac{\omega\mu_0}{2\sigma}} \quad (ohms)$$ Where P is perimeter of the cross section of rod ($P = C = 2\pi b\ for\ a\ circular\ wire\ of\ radius\ b$) R_s is the conductor surface resistance ω angular frequency μ_0 permeability of free space σ conductivity of the metal
Loss resistance R_L (*straight wire/λ/2 dipole*)	$$R_L = \frac{l}{2P}\sqrt{\frac{\omega\mu_0}{2\sigma}}$$
Maximum gain G_0	$$G_0 = e_{cd}\, D_{max} = e_{cd}\, D_0$$
Partial gain G_θ, G_ϕ	$$G_0 = G_\theta + G_\phi$$ $$G_\theta = \frac{4\pi U_\theta}{P_{in}}\ , G_\phi = \frac{4\pi U_\phi}{P_{in}}$$
Absolute gain G_{abs}	$$G_{abs} = e_r\, G(\theta,\phi) = e_r\, e_{cd}\, D(\theta,\phi) = (1 - \lvert\Gamma\rvert^2)\, e_{cd}\, D(\theta,\phi) = e_0\, D(\theta,\phi)$$
Beam efficiency BE	$$BE = \frac{\int_0^{2\pi}\int_0^{\theta_1} U(\theta,\phi)\ \sin\theta\, d\theta\, d\phi}{\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi}$$
Polarization loss factor (PLF)	$$PLF = \lvert\hat{\boldsymbol{\rho}}_w \cdot \hat{\boldsymbol{\rho}}_{\boldsymbol{a}}\rvert^2 = \lvert\cos\psi_p\rvert^2 \quad (dimensionless)$$ $\hat{\boldsymbol{\rho}}_w$ is unit vector of the wave $\hat{\boldsymbol{\rho}}_{\boldsymbol{a}}$ is polarisation vector ψ_p angle between two unit vector
Vector effective length $l_e(\theta,\phi)$	$$l_e(\theta,\phi) = \hat{a}_\theta\, l_\theta(\theta,\phi) + \hat{a}_\phi\, l_\phi(\theta,\phi)$$
Polarization efficiency p_e	$$p_e = \frac{\lvert l_e \cdot \mathbf{E}^{inc}\rvert^2}{\lvert l_e\rvert^2\, \lvert\mathbf{E}^{inc}\rvert^2}$$ l_e vector effective length of the antenna $\mathbf{E}^{inc}$ incident electric field
Antenna impedance Z_A	$$Z_A = R_A + jX_A = (R_r + R_L) + jX_A$$ Where Z_A antenna impedance at terminal a-b (ohms) R_A antenna resistance at terminal a-b (ohms) X_A antenna reactance at terminal a-b (ohms)

Aperture efficiency ε_{ap}	$$\varepsilon_{ap} = \frac{A_{em}}{A_p} = \frac{maximum\ effective\ area}{physical\ area}$$
Radar cross section (RCS)	$$\sigma = \lim_{R\to\infty}\left[4\pi R^2 \frac{W_s}{W_i}\right] = \lim_{R\to\infty}\left[4\pi R^2 \frac{\lvert\mathbf{E}^s\rvert^2}{\lvert\mathbf{E}^i\rvert^2}\right] = \lim_{R\to\infty}\left[4\pi R^2 \frac{\lvert\mathbf{H}^s\rvert^2}{\lvert\mathbf{H}^i\rvert^2}\right]$$
Brightness temperature $T_B(\theta,\phi)$	$$T_B(\theta,\phi) = \varepsilon(\theta,\phi)T_m = (1-\lvert\Gamma\rvert^2)T_m$$
Antenna temperature T_A	$$T_A = \frac{\int_0^{2\pi}\int_0^{\pi} T_B(\theta,\phi)\,G(\theta,\phi)\,\sin\theta\,d\theta\,d\phi}{\int_0^{2\pi}\int_0^{\pi} G(\theta,\phi)\sin\theta\,d\theta\,d\phi}$$

Radiation Integrals and Auxiliary Potential Functions

Parameter	Formula
Vector potential **A** for an electric current source **J**	$$\mathbf{E}_A = -\nabla\phi_e - j\omega\mathbf{A} = -j\omega\mathbf{A} - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla\cdot\mathbf{A})$$
Vector potential **F** for a magnetic current source **M**	$$\mathbf{H}_F = -j\omega\mathbf{F} - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla\cdot\mathbf{F})$$
Total field	$$\mathbf{E} = \mathbf{E}_A + \mathbf{E}_F = -j\omega\mathbf{A} - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla\cdot\mathbf{A}) - \frac{1}{\epsilon}\nabla\times\mathbf{F}$$ or $$\mathbf{E} = \mathbf{E}_A + \mathbf{E}_F = \frac{1}{j\omega\epsilon}\nabla\times\mathbf{H}_A - \frac{1}{\epsilon}\nabla\times\mathbf{F}$$ and $$\mathbf{H} = \mathbf{H}_A + \mathbf{H}_F = \frac{1}{\mu}\nabla\times\mathbf{A} - j\omega\mathbf{F} - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla\cdot\mathbf{F})$$ **or** $$\mathbf{H} = \mathbf{H}_A + \mathbf{H}_F = \frac{1}{\mu}\nabla\times\mathbf{A} - \frac{1}{j\omega\mu}\nabla\times\mathbf{E}_F$$

Linear Wire Antennas

Parameter	Formula
	Infinitesimal Dipole
Normalized power pattern	$$U = (E_{\theta n})^2 = C_0 sin^2\theta$$

Wave impedance Z_w	$Z_w = \frac{E_\theta}{H_\phi} \simeq \eta = 377\ ohms$
Vector effective length l_e	$l_e = -\hat{a}_\theta l \sin\theta$ $\lvert l_e \rvert_{max} = \lambda$
Half-power beamwidth	$\text{HPBW} = 90^0$
Loss resistance R_L	$R_L = \frac{l}{P}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{l}{2\pi b}\sqrt{\frac{\omega\mu_0}{2\sigma}}$
Small Dipole	
Normalized power pattern	$U = (E_{\theta n})^2 = C_1\, sin^2\,\theta$
Vector effective length l_e	$l_e = -\hat{a}_\theta \frac{l}{2}\sin\theta$ $\lvert l_e \rvert_{max} = \frac{l}{2}$
Half-power beamwidth	$\text{HPBW} = 90^0$
Half Wavelength Dipole	
Normalized power pattern	$U = (E_{\theta n})^2 = C_2\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2 \simeq C_2 sin^3\,\theta$
Wave impedance Z_w	$Z_w = \frac{E_\theta}{H_\phi} \simeq \eta = 377\ ohms$
Vector effective length l_e	$l_e = -\hat{a}_\theta \frac{\lambda}{\pi}\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$ $\lvert l_e \rvert_{max} = \frac{\lambda}{\pi} = 0.3183\lambda$
Half-power beamwidth	$\text{HPBW} = 78^0$
Loss resistance R_L	$R_L = \frac{l}{P}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{l}{4\pi b}\sqrt{\frac{\omega\mu_0}{2\sigma}}$
Quarter-Wavelength Dipole	
Normalized power pattern	$U = (E_{\theta n})^2 = C_2\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2 \simeq C_2 sin^3\,\theta$

Wave impedance Z_w	$Z_w = \frac{E_\theta}{H_\phi} \simeq \eta = 377\ ohms$
Vector effective length l_e	$l_e = -\hat{a}_\theta \frac{\lambda}{\pi} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$ $\lvert l_e \rvert_{max} = \frac{\lambda}{\pi} = 0.3183\lambda$

Loop Antennas

Parameter	Formula
	Small circular loop
Normalized power pattern	$U = \lvert E_{\phi n} \rvert^2 = C_0 sin^2\,\theta$
Wave impedance Z_w	$Z_w = -\frac{E_\phi}{H_\theta} \simeq \eta = 377\ ohms$
Loss resistance R_L (one turn)	$R_L = \frac{l}{P}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{l}{2\pi b}\sqrt{\frac{\omega\mu_0}{2\sigma}}$
Loss resistance R_L (N turns)	$R_L = \frac{N\,a}{b}\ R_s\left(\frac{R_p}{R_0} + 1\right)$ Where R_s surface impedance of conductor = $\sqrt{\frac{\omega\mu_0}{2\sigma}}$ R_p ohmic resistance per unit length due to proximity effect R_0 ohmic skin effect resistance per unit length = $\frac{N\,R_s}{2\pi b}$ $(ohms/m)$ a circular loop of radius b wire radius
Loop external inductance (L_a)	$L_a = \mu_0 a\left[ln\left(\frac{8a}{b}\right) - 2\right]$
Loop internal inductance (L_i)	$L_i = \frac{a}{\omega b}\sqrt{\frac{\omega\mu_0}{2\sigma}}$
Vector effective length l_e	$l_e = \hat{a}_\phi jk_0\pi a^2 \cos\psi_i\ sin\theta_i$ Where ψ_i is angle between the direction of magnetic field of the incident field and the plane of incidence
Half-power beamwidth	HPBW $= 90^0$
Large circular loop **(Uniform current)**	
Normalized power pattern	$U = \lvert E_{\phi n} \rvert^2 = C_1\, J_1{}^2\,(ka\sin\theta)$
Wave impedance Z_w	$Z_w = -\frac{E_\phi}{H_\theta} \simeq \eta = 377\ ohms$

Loss resistance R_L (one turn)	$R_L = \frac{l}{P}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{l}{2\pi b}\sqrt{\frac{\omega\mu_0}{2\sigma}}$ Where l is length P perimeter (circumference) of the wire loop
Loss resistance R_L (N turns)	$R_L = \frac{N_a}{b}\ R_s\left(\frac{R_p}{R_0}+1\right)$
External inductance L_A	$L_A = \mu_0 a\left[ln\left(\frac{8a}{b}\right)-2\right]$
Internal inductance L_i	$L_i = \frac{a}{\omega b}\sqrt{\frac{\omega\mu_0}{2\sigma}}$
Vector effective length l_e	$l_e = \hat{a}_{\Phi} jk_0\pi a^2 \cos\psi_i\ sin\theta_i$
Small Square Loop **(Uniform current, a on Each Side)**	
Normalized power pattern (principal plane)	$U = \left\|E_{\Phi n}\right\|^2 = C_2\ sin^2\ \theta$
Wave impedance Z_w	$Z_w = -\frac{E_\phi}{H_\theta} \simeq \ \eta = 377\ ohms$
Loss resistance R_L	$R_L = \frac{4a}{P}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{4a}{2\pi b}\sqrt{\frac{\omega\mu_0}{2\sigma}}$
External inductance L_A	$L_A = 2\ \mu_0\frac{a}{\pi}\left[ln\left(\frac{a}{b}\right)-0.774\right]$
Internal inductance L_i	$L_i = \frac{4a}{\omega b}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{4a}{2\pi b\omega}\sqrt{\frac{\omega\mu_0}{2\sigma}}$
Ferrite Circular Loop (**uniform current**)	
Demagnetizing factor D	Ellipsoid: $D = \left(\frac{a}{l}\right)^2\left[\ln\left(\frac{2l}{a}\right)-1\right]$ $l \gg a$ Sphere: $D = \frac{1}{3}$ Where a is radius of ellipsoid l length of ellipsoid

Arrays: Linear, Planar and Circular

Parameter	Formula
	Dolph-Tschebyscheff Array
Array factor	$m = 0\ \cos(mu) = 1$ $m = 1\ \cos(mu) = \cos u$ $m = 2\ \cos(mu) = \cos(2u) = 2\,cos^2 u - 1$ $m = 3\ \cos(mu) = \cos(3u) = 4\,cos^3 u - 3\cos u$ $m = 4\ \cos(mu) = \cos(4u) = 8\,cos^4 u - 8\,cos^2 u + 1$ $m = 5\ \cos(mu) = \cos(5u) = 16\,cos^5 u - 20\,cos^3 u + 5\cos u$ $m = 6\ \cos(mu) = \cos(6u) = 32\,cos^6 u - 48\,cos^4 u + 18\,cos^2 u - 1$ $m = 7\ \cos(mu) = \cos(7u) = 64\,cos^7 u - 112\,cos^5 u + 56\,cos^3 u - 7\cos u$ $m = 8\ \cos(mu) = \cos(8u) = 128\,cos^8 u - 256\,cos^6 u + 160\,cos^4 u - 32\,cos^2 u + 1$ $m = 9\cos(mu) = \cos(9u) = 256cos^9 u - 576\,cos^7 u + 432cos^5 u - 120\,cos^3 u + 9\cos u$ *The above are obtained by the use of Euler's formula* $[e^{ju}]^m = (\cos u + j\sin u)^m = e^{jmu} = \cos(mu) + j\sin(mu)$ Then $m = 0\ \cos(mu) = 1 = T_0(z)$ $m = 1\ \cos(mu) = z = T_1(z)$ $m = 2\ \cos(mu) = 2z^2 - 1 = T_2(z)$ $m = 3\ \cos(mu) = 4z^3 - 3z = T_3(z)$ $m = 4\ \cos(mu) = 8z^4 - 8z^2 + 1 = T_4(z)$ $m = 5\ \cos(mu) = 16z^5 - 20z^3 + 5z = T_5(z)$ $m = 6\ \cos(mu) = 32z^6 - 48z^4 + 18z^2 - 1 = T_6(z)$ $m = 7\ \cos(mu) = 64z^7 - 112z^5 + 56z^3 - 7z = T_7(z)$ $m = 8\ \cos(mu) = 128z^8 - 256z^6 + 160z^4 - 32z^2 + 1 = T_8(z)$ $m = 9\ \cos(mu) = 256\,z^9 - 576z^7 + 432z^5 - 120z^3 + 9z = T_9(z)$