Question Paper

Exam Date & Time: 06-Jul-2022 (09:00 AM - 12:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

SECOND SEMESTER B.TECH. EXAMINATIONS (MIT MANIPAL) - JUNE/JULY 2022
SUBJECT : MAT 1251 - ENGINEERING MATHEMATICS-II
(PHYSICS AND CHEMISTRY GROUP)

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

1A) Find the possible percentage error in computing the resistance r from the formula $\frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}$, if r_1 and r_2 are both in error by 2%. (3)

1B) Expand $f(x,y)=e^y log_e(1+x)$ in powers of x and y up to the third degree terms. (3)

1C) If $V=r^m$, where $r^2=x^2+y^2+z^2$, then prove that $V_{xx}+V_{yy}+V_{zz}=m\,(m+1)\,r^{m-2}$. (4)

2A) Find $\lim_{x\to 0}\frac{(1+x)^{\frac{1}{x}}-e+\frac{ex}{2}}{x^2}$ (3)

2B) Using Lagrange's method of undetermined multipliers, find the extreme values of the function $f(x,y,z)=xy^2z^3$ subject to the condition x+y+z=24. (3)

2C) Change the order of integration and evaluate $\int_0^1\int_{\sqrt{y}}^{2-y} xy\,dxdy$. (4)

3A) Test the convergence of the series $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\frac{7}{4.5.6}+\cdots\infty$ (3)

3B) Find the area common to the circles, $r=2\sin\theta$ and $r=2\cos\theta$ using double integration. (3)

3C) Find the centre, radius and area of the circle (4)

$x^2+y^2+z^2-2y-4z=11,\ x+2y+2z=15.$

4A) Evaluate the integral $\iiint(x^2+y^2+z^2)dx\,dy\,dz$ over the volume enclosed by the sphere $x^2+y^2+z^2=1$. (3)

4B) (3)

Find $L^{-1}\left[\log\left(\frac{s(s-2)}{s^2+9}\right)\right]$.

4C) Solve the differential equation using Laplace transform (4)

$y''' + 2y'' - y' - 2y = 0$, $y(0) = 0$, $y'(0) = 0$ and $y''(0) = 6$.

5A) Express the function $f(t) = \begin{cases} \cos t \,; 0 < t < \pi \\ \sin t \,; \quad t > \pi \end{cases}$ (3)

in to unit step function and hence find its Laplace transform.

5B) Test the convergence of the infinite series $1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \frac{5!}{5^5} + \cdots \infty$ (3)

5C) Using beta and gamma functions, prove that (4)

$$\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\; d\theta \times \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$$

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