## **Question Paper**

Exam Date & Time: 14-Jun-2022 (02:00 PM - 05:00 PM)

MANIPAL INSTITUTE OF TECHNOLOGY MANIPAL

(A constituent unit of MAHE, Manipal)

IV Semester End Semester Examination Engineering Mathematics IV (MAT 2253)

ENGINEERING MATHEMATICS IV [MAT 2253]

Marks: 50

Duration: 180 mins.

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Section Duration: 180 mins

**Descriptive Questions** 

## Answer all the questions.

1)

2)

A)

$$f(x) = \begin{cases} ax, & 0 \le x \le 2\\ (4-x)a, & 2 \le x \le 4\\ 0, & otherwise \end{cases}$$

i) find the value of a, ii) find the CDF of X, and iii) find P(X>2.5) .

If the probability density function of a continuous random variable X is given by

- B) The three bags contain 6 red, 4 black balls; 4 red, 6 black balls; 5 red, 5 black balls respectively. One of the bags is selected at (3) random and a ball is drawn from it. If the ball drawn is red, find the probability that it is drawn from the first bag.
- C) Three group of children contain respectively 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 3 boys. One child is selected at (3) random from each group. Find the chances of selecting 1 girl and 2 boys.

A 2D random variable has joint PDF

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4)

A)

$$f(x,y) = \begin{cases} x^2 + \frac{3}{3}, & 0 < x < 1, & 0 < y < 2\\ 0, & elsewhere \end{cases}$$

xy

Compute E(X) and V(Y).

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B) Using method of least squares, fit a curve of the form  $y = ab^x$  for the following data. (3)

х	2	3	4	5	6
у	144	172.8	207.8	248.8	298.5

- C) An irregular six faced dice is thrown 12 times. The probability that it will give six even number is twice the probability that it will (3) give five even numbers. If 1000 sets, each of exactly 12 trials are made, how many sets are expected not to give any even number?
- 3) In an examination, the candidates are awarded the following grades depending on the marks scored by them: distinction: (4)
  - A) 80%, first class:  $60\% \le marks < 80\%$ , second class:  $45\% \le marks < 60\%$ ; third class:  $30\% \le marks < 45\%$ ; fail: < 30%. It

was found that 8% of the students failed and 8% have scored distinction. Find the average marks obtained by the candidates.

Deduce the percentage of students placed in second class. Assume normal distribution of marks.

- B) In a book of 600 pages, there are 60 typographical errors. Assuming Poisson law for the number of errors per page, find the probability that a randomly chosen 4 pages will contain no errors.
- C) Apply graphical method to find the maximum value of  $Z=5x_1+3x_2^{ ext{ subject to}}$

$$4x_1 + 5x_2 \le 1000; 5x_1 + 2x_2 \le 1000; 3x_1 + 8x_2 \le 1200; x_1 \ge 0, x_2 \ge 0$$

Use simplex method to minimize 
$$Z = x_1 - 3x_2 + 3x_2$$
, subject to

(5)

(3)

A)

$$3x_1 + x_2 + 2x_3 \le 7$$
  

$$2x_1 + 4x_2 \ge -12$$
  

$$-4x_1 + 3x_2 + 8x_3 \le 10,$$
  

$$x_1, x_2, x_3 \ge 0.$$

 $3v_{-} - v_{-} \pm 2v_{-} < 7$ 

B) Consider the following linear programming problem:

$$\max Z = 12x_1 + 8x_2 + 14x_3 + 10x_4$$

subject to

$$5x_1 + 4x_2 + 2x_3 + x_4 = 100$$

$$2x_1 + 3x_2 + 8x_3 + x_4 = 75,$$

find a) basic feasible solution, b) non-degenerate basic feasible solution, and c) optimal basic feasible solution.

A person wants to decide the constituents of a diet which will fulfil his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yields per unit of these foods are given in the following table:

Food Type	Yield per unit			Cost/unit (₹)
	Proteins	Fats	Carbohydrates	
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Minimum requirement	800	200	700	

Formulate the linear programming model for the problem.

Find the series solution near of the differential equation

$$x = 0$$
  $9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0.$  (4)

is the Bessel's function.

A)

B)

C)

5)

C)

Prove that  $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)^{\text{, where}} J_n(x)^{\text{If }}$ 

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$$

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