Exam Date & Time: 25-Jul-2022 (02:00 PM - 05:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

ENGINEERING MATHEMATICS IV [MAT 2257]

Marks: 50 **Duration: 180 mins.**

Descriptive Questions

Answer all the questions. Section Duration: 180 mins

1) Solve the following PDE $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=-81xy,\ 0<x<1, 0<y<1,$

A) $u(0,y)=u(x,0)=0\ and\ u(1,y)=u(x,1)=100$ taking $h=\frac{1}{3}$. (3)

B) Solve $y''+xy=1,\ \ y(0)=0,\ y'(1)=1$ with $h=0.5$ by finite difference method. (3)

C) Solve the wave equation $u_{tt}=u_{xx},\ 0<x<1, t>0, u(x,0)=\frac{\partial u}{\partial t}(x,0)=0$,

$u(0,t)=0$ and $u(1,t)=100\sin \pi t$. Compute u for 4-timesteps with $h=\frac{1}{4}$. (4)

2) Fit a curve $y=ax^2+bx+c$ for the following data by the method of

A)

x	-1	0	1	2
y	-2	1	2	4

principle of least square . (3)

B) Solve the difference equation using Z-transform method

$y_{n+2}-3y_{n+1}-10y_n=0, y_1=0, y_0=1$. (3)

C) Solve the difference equation $y_{n+2}-7y_{n+1}-8y_n=2^n[n]^2$. (4)

3) A survey shows 56% of all residents in a city have a workplace retirement plan, 68% Have health insurance, and 49% have both benefits. At random one worker is selected. (3)

A)

a. What is the probability that he has neither health insurance nor a retirement plan?

b. What is the probability that he has health insurance if he has the retirement plan?

c. Are having health insurance and a retirement plan independent events? Explain .

B) Suppose the pdf of a random variable X is given as

$$f(x)=\begin{cases}\frac{1-x}{2}, & -1<x<1\\ 0, & otherwise\end{cases}$$ (3)

Find $E(X)$ and $V(X)$. .

C) The contents of urns I, II, III are as follows:

1 white, 2 black and 3 red balls,

2 white, 1 black and 1 red balls and 4 white, 5 black and 3 red balls respectively. One urn is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they come from urns I, II and III?. (4)

4) Two dimensional random variables X and Y have joint pdf

A) $f(x,y)=\begin{cases}x+y, & 0\le x\le 1, 0\le y\le 1\\ 0 & elsewhere\end{cases}$ find the correlation coefficient between X and Y (3)

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B) If 5% of the electric bulbs manufactured by a company are defective, use Poisson distribution to find the probability that in a sample of 100 bulbs

i. none is defective (3)

ii. 5 bulbs will be defective .

C) The joint pdf of two continuous random variables X and Y is given as:

$$f(x,y)=\begin{cases}c(2x+y), & 2<x<6, 0<y<5\\ 0, & otherwise\end{cases}$$

a. What is the value of c? (4)

b. Write the marginal pdfs for X and Y.

c. Find $P(X+Y>4)$.

5) Find the moment generating function of exponential distribution and hence find its mean and variance . (3)

A)

B) Suppose that X is uniformly distributed over the interval $(-1,1)$. Obtain the pdf of the random variable $Y = \sin\left(\frac{\pi X}{2}\right)$. (3)

C) In an examination, the marks scored by the students follows the normal distribution. It is known that a student passes the examination if he secures 40% or more marks; he is placed in first, second and third division, if he secures 60% more, between 50% and 60% more, between 40% and 50% marks, respectively. He gets a distinction, if he gets 70% or more marks. It is given that 10% of the students have failed in the examination and 5% of them obtained distinction. Find the percentage of students getting second division . (4)

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