Exam Date & Time: 16-Jun-2022 (09:00 AM - 12:00 PM)

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

**ENGINEERING MATHEMATICS IV [MAT 2257]**

**Marks: 50** **Duration: 180 mins.**

**Descriptive Questions**

**Answer all the questions.** Section Duration: 180 mins

1) Use the Bender-Schmidt method to solve the following equation: $\frac{\partial u}{\partial t} = \frac{1}{2}\frac{\partial^2 u}{\partial x^2}$

A) With respect to the conditions: $u(x,0) = 4x - x^2, u(0,t) = u(4,t) = 0$ taking $h = 1$. Compute u for three time steps. (4)

B) solve $u_{xx} + u_{yy} = 0$, $0 < x < 1,\ 0 < y < 1$, given $u(0,y) = u(x,0) = 0$, $u(1,y) = u(x,1) = 50$ taking h=1/3 . (3)

C) Solve the boundary value problem $y'' + y + 1 = 0,\ y(0) = y(1) = 0,$ using the method of finite differences by taking width $h = \frac{1}{4}$ . (3)

2) Solve the difference equation $y_{n+2} - 4y_n = n^2 + n - 1$ . (4)

A)

B) Fit a curve $y = ab^x$ for the following data by the method of principle of least square (3)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| y | 1 | 1.2 | 1.8 | 2.5 | 3.6 | 4.7 | 6.6 | 9.1 |

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C) Solve the Simultaneous equations using Z-transform method $x_{n+1} - y_n = 1,$ $y_{n+1} + x_n = 0, x_0 = 0, y_0 = -1$ . (3)

3) Suspose three companies X, Y and Z produce televisions. X produce twice as many as (4)

Y,

A) while Y and Z produce the same number. It is known that 2% of X, 2% of Y and 4% of Z

are defective. All the televisions produced are put into one shop and then one television

is selected at random.

i. What is the probability that the television is defective?

ii. Suppose a television chosen is defective, what is the probability that this television is produced by the company X? .

B) A continuous random variable X has a pdf $f(x) = kx^2e^{-x},\ x \geq 0.$ Find the value of

k and also find variance of X. (3)

C) Two absent minded roommates forget their umbrellas in some way or another. A

always takes his umbrella when he goes out, while B forgets to take his umbrella with

probability 1/2. Each of them forgets his umbrella at a shop with probability 1/4. After

visiting 3 shops they return home. Find the probability that (3)

i. They have both the umbrellas

ii. They have only one umbrella.

4) The joint pdf of two continuous random variables X and Y is given as:

A) $$f(x,y) = \begin{cases} (x^2 + \frac{xy}{3}), & 0 \leq x \leq 1, 0 \leq y \leq 2 \\ 0, & otherwise \end{cases}$$ (4)

Find i) The marginal pdfs for X and Y. ii) $P(X+Y \geq 1)$ iii) $P\left(Y < \frac{1}{2} | X < \frac{1}{2}\right)$.

B) If $Var(X+2Y) = 40$ and $Var(X-2Y) = 20,$

i. What is $Cov(X,Y)?$

ii. If $Var(X) = 2Var(Y),$ what is the correlation coefficient $\rho_{XY}?$ . (3)

C) If the probability that an individual will suffer a bad reaction from injection of a given

serum is $0.001$ , determine the probability using Poisson's distribution that out of (3)

2000 individuals

i) Exactly 3 individuals ii) More than 2 individuals will suffer a bad reaction.

5) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. (4)

A)

B) Let $X_1$, $X_2$ and $X_3$ are independent random variables with $M_{X_1}(t) = e^{2t(1+t)}$, $M_{X_2}(t) = e^{3t(1+t)}$, $M_{X_3}(t) = e^{4t(1+t)}$. Then find the pdf of $Z = 4X_1 + X_2 + 2X_3$. (3)

Hence find E(Z/2) and V(Z/2) .

C) If the continuous random variable X is uniformly distributed in (-2,2) , find the pdf of $Y = 6 - X^2$. (3)

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