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# MANIPAL INSTITUTE OF TECHNOLOGY

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**FOURTH SEMESTER B.TECH. (AUTOMOBILE ENGINEERING)**
**END SEMESTER EXAMINATIONS, JUNE 2022**

SUBJECT: ENGINEERING MATHEMATICS IV [MAT 2252]

Time: 3 Hours MAX. MARKS: 50

**Instructions to Candidates:**

- Answer **ALL** the questions.
- Missing data may be suitably assumed.

| | | |
|---|---|---|
| **1A.** | In general when A and B play 12 games of chess, 6 are won by A, 4 are won by B and 2 end in a draw. They agree to play a tournament consisting of 3 games. Find the probability that i) 2 games end in a draw ii) A and B win alternatively, iii) B wins atleast one game. | **3M** |
| **1B.** | Find the mean and variance of Gamma distribution by defining its probability distribution function. | **3M** |
| **1C.** | Suppose that life length of two electronic device say $D_1$ and $D_2$ have distributions $N(40,36)$ and $N(45,9)$ respectively. If the electronic device is to be used for 45 hours period, which device is to be preferred? If it is to be used for 48 hours period, which device is to be preferred? | **4M** |
| **2A.** | The joint density function of (x, y) is given by $f(x,y)=\begin{cases}\frac{15}{12}x(2-x-y), & 0\le x\le 1,\ 0\le y\le 1\\ 0, & \textit{elsewhere}\end{cases}$ Compute conditional joint probability density function of x given y. | **3M** |
| **2B.** | Find the mean and variance of a random variable A with the pdf $f(x) = xe^{\frac{-x^2}{2}}$, X>0. | **3M** |
| **2C.** | Fit a parabola for the following data and estimate the value of y at x=3.5. | **4M** |
| **3A.** | If a random variable k is uniformly distributed over (0, 5), what is the probability that the roots of the equation $4x^2 + 4xk + k + 2 = 0$ are real? | **3M** |
| **3B.** | Solve using Graphical method<br>$\textit{Maximize } z = 10x + 15y$<br>$\textit{Subject to } 2x + y \le 26$<br>$2x + 4y \le 56$<br>$-x + y \le 5$<br>$x,\ y \ge 0$ | **3M** |

| **x** | **1** | **2** | **3** | **4** | **6** | **8** |
|---|---|---|---|---|---|---|
| **y** | **2.4** | **3** | **3.6** | **4** | **5** | **6** |

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| **3C.** | Calculate the mean of the following data relating to weight of 120 articles.<br><br>Weight (in grams): 0 -10 \| 10 – 20 \| 20 – 30 \| 30 – 40 \| 40 – 50 \| 5 - 60<br>No of articles: 14 \| 17 \| 22 \| 26 \| 23 \| 18 | **4M** |
| **4A.** | In a bolt manufacturing factory, there are 3 machines A, B, C. The machine produce 25%, 35%, and 40% respectively of the total product. A bolt is chosen at random from product and found to be defective. It is believed that 5%, 4%, and 2% respectively of the products manufactured by A, B, C are usually defective. What is the probability that the defective bolt selected was manufactured by machine C? | **3M** |
| **4B.** | Two independent random variable X and Y have mean 6 and 9 and variance 16 and 25 respectively. Find the correlation coefficient between U = 4X-3Y and V = X+3Y. | **3M** |
| **4C.** | If X is a random variable taking values 0, 1, 2,…. and $P(X) = ab^x$, where a and b are positive constants such that a+b = 1, then (i) Find mgf. (ii) If $E(X) = m_1, E(X^2) = m_2$, show that $m_2 = m_1(2m_1 + 1)$. | **4M** |
| **5A.** | If $X \sim N(\mu, \sigma^2)$, then show that $Z = \frac{X-\mu}{\sigma} \sim N(0,1)$ and $Y = Z^2 \sim \chi^2(1)$. | **3M** |
| **5B.** | Let $\bar{X}$ and $s^2$ be the mean and variance of a random sample of size 25 from the distribution N (3,100). Evaluate $Pr\{0 < \bar{X} < 6 \ , 55.2 < s^2 < 145.6\}$. | **3M** |
| **5C.** | Use Big M method to solve<br><br>$Maximize\ P = -3x + y + z$<br>$Subject\ to\ x - 2y + z \le 11$<br>$-4x + y + 2z \ge 3$<br>$2x - y = -1$<br>$x,\ y,\ z \ge 0$ | **4M** |