## Exam Date & Time: 11-Jun-2022 (09:00 AM - 12:00 PM)



## IV Semester End Semester Examination ENGINEERING MATHEMATICS - IV [MAT 2260]

**Duration: 180 mins.** 

## **Descriptive Questions**

## Answer all the questions.

Section Duration: 180 mins

- 1) Box *A* contains 3 red and 2 white ball, box *B* contains 2 red and 5 white balls. A box is selected at random. A ball is drawn and put into the other. Then a ball is drawn from that box. Find the probability that both balls are of same colour. (3)
  - A)

Marks: 50

B) If *X* is a continuous random variable with p. d. f. given by

$$f(x) = \begin{cases} ax, & 0 \le x \le 1\\ a, & 1 \le x \le 2\\ 3a - ax, & 2 \le x \le 3\\ 0, & Otherwise \end{cases}$$
(3)

(a) Find the constant k. (b) Find the c. d. f. of X.

C) Three news papers A, B and C publish in a city. Recent survey indicates the following.
20% read A, 16% read B, 14% read C, 8% read A and B, 5% read A and C, 4% read B and C, 2% read A, B and C. One person is chosen at random. Find the probability that he reads

(i)None of the papers.

(ii)Reads  $^{A}$  and  $^{B}$  if it is known that he reads at least one.

(iii)Exactly one paper.

- 2) Find the correlation coefficients between x and y when the lines of regression are 2x 9y + 6 = 0 and x 2y + 1 = 0. (3)
  - A)
  - B) Fit a least square quadratic approximation for the following data

x	1	2	3	4	5	6	7
у	80	90	92	83	94	99	92

(3)

(4)

	C)	(i) The number of patients requiring ICU in a hospital is a random variable with mean <b>18</b> and standard deviation <b>2.5</b> . Determine the minimum probability that the number of patients requiring ICU is between <b>8</b> and <b>28</b> ?							
		(ii) Two aeroplanes bomb a target in succession. The probability of each correctly attaining a hit is <b>0.3</b> and <b>0.2</b> respectively. The second aeroplane will bomb only if the first one will miss the target. Find the probability that target is hit?	(4)						
3)	A)	A target is to be destroyed in a bombing exercise. There is 75% chance that any one bomb will strike the target. Assume that two direct hits are required to destroy the target completely. How many minimum bombs must be dropped in order that the chances of destroying the target is $\geq$ 99%?	(3)						
	B)	In a Normal distribution 31% items are below 45 and 8% are over 64. Find mean and standard deviation of the distribution.	(3)						
	C)	The joint PDF of the random variable X, and Y is given by							
		$f(x, y) = k(xy + y^{2}),  0 \le x \le 1, \qquad 0 \le y \le 2$ Find (i) $P(Y > 1)$ (ii) $P\left(X > \frac{1}{2}, Y < 1\right)$ (iii) $P(X + Y \le 1)$							
4)		If X has Cauchy's distribution with pdf $f(x) = \frac{1}{\pi(1+x^2)}$ then show that $Y = \frac{1}{x}$ also has Cauchy's distribution.	(3)						
	A)	- <del>V</del>							
	В)	Let $\bar{x}$ be mean of random sample of size 5 taken from Normal distribution of $\mu = 0$ , $\sigma^2 = 125$ . Determine c so that $P\{\bar{X} < c\} = 0.9$ .	(3)						
	C)	Find the moment generating function of a random variable which is uniformly distributed							
		in $-a < x < a$ . Hence evaluate $E(x^{2n})$ .	(4)						
5)		Prove that $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$ .	(3)						
	A) B)	Prove that $P_n(x) = \frac{1}{n! 2^n} \frac{d^n}{dx^n} (x^2 - 1)^n$ .	(3)						

(4)

C) Find the Frobenius series solution of 
$$8x^2y'' + 10xy' - (1+x)y = 0$$
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