Exam Date & Time: 20-Jul-2022 (02:00 PM - 05:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

IV Semester End Semester Make-Up Examination

ENGINEERING MATHEMATICS - IV [MAT 2260]

Marks: 50 **Duration: 180 mins.**

Descriptive Questions

Answer all the questions. Section Duration: 180 mins

1) Let A and B are two events associated with an experiment. Given $P(A) = 0.4$, $P(A \cup B) = 0.7$ and $P(B) = p$. What choice of p

A) (i) A and B are independent (3)

(ii) A and B are mutually exclusive

B) Let X be the discrete random variable taking values $1,2,3,\ldots,\infty,$ and $P(X = j) = \frac{1}{2^j}$, where $j = 1,2,3,\ldots,\infty.$ Find

(i) Probability of X is even

(3)

(ii) Probability of X is odd

(iii) Probability of X is divisible by $3.$

C) In a bolt factory, machines A, B and C manufactures $25\%, 35\%$ and 40% of the total output respectively. Of their outputs, $5\%, 4\%$ and 2% respectively, are defective bolts. A bolt is chosen at random and found to be defective. What is the probability that the bolt came from (4)

(i) Machine A ? (ii) Machine B ? (iii) Machine C ?.

2) The regression equations calculated from a given set of observation for two random variables are x = -0.4y + 6.4; y = -0.6x +4.6. Calculate

A) (i) Mean of x (3)

(ii) Mean of y

(iii) Correlation coefficient between x and y.

B) Using method of least squares, fit a curve of the form y = a + bx for the following data: (3)

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

C) The diameter of an electric cable say X is assumed to be a continuous random variable with p. d. f. $f(x) = \begin{cases} 6x(1-x) & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$.

i. Obtain expression for c. d. f.

ii. Find b, such that $P(X < b) = 2P(X > b)$ (4)

iii. Compute $P\left(X \le \frac{1}{2} | \frac{1}{3} < X < \frac{2}{3}\right)$.

3) An irregular six faced dice is thrown 12 times. The probability that it will give six even number is twice the probability that it will give five even numbers. If 1000 sets, each of exactly 12 trials are made, how many sets are expected not to give any even number? (3)

A)

B) In a Normal distribution 7% items are under 35 and 89% are under 63. Find mean and standard deviation of the distribution. (3)

C) The joint probability mass function of (X, Y) given by $P(x, y) = k(3x + 5y),\ x = 1, 2, 3;\ y = 0, 1, 2.$ Find

(i) k

(ii) marginal distributions of X and Y. (4)

(iii) $P(X \le 2 \,|Y \le 1)$.

4) Find the pdf of $y = -2log_e x$ where $f(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & elsewhere \end{cases}$ (3)

A)

B) Let $\overline{X}$ be mean of random sample of size n taken from Normal distribution $N(\mu, 100)$ (3)

Find n so that $P\{\mu - 5 < \overline{X} < \mu + 5\} = 0.954$.

C) Find the Mgf of Poisson's Distribution. Hence find Mean and variance of Poisson's distribution. (4)

5) Prove that $\frac{d}{dx}(x^{-n} J_n(x)) = -x^n J_{n+1}(x)$. (3)

A)

B) Prove that $\int_{-1}^{1} P_m(x)\, P_n(x)\, dx = 0$, when $m \neq n$. (3)

C) Find the Frobenius series solution of $9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0.$ (4)

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