Question Paper

Exam Date & Time: 20-Jul-2022 (02:00 PM - 05:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

IV Semester Makeup End Semester Examination

ENGINEERING MATHEMATICS - IV [MAT 2261]

Marks: 50 **Duration: 180 mins.**

Descriptive Questions

Answer all the questions. Section Duration: 180 mins

1) A) In a college where boys and girls are in equal in proportion, it was found that 10 out of 100 boys and 25 out of 100 girls were using the same brand of a two wheeler. If a student using that was selected at random what is the probability of being boy? (3)

B) Given $f(x) = \begin{cases} kx^3 ; 0 < x < 1 \\ 0 \quad ; elsewhere \end{cases}$ (3)

i. Find k such that $f(x)$ is a valid probability distribution function (pdf)

ii. Find the cumulative distribution function (cdf) of X.

iii. Find $P\left(X < \frac{1}{2}\right)$.

C) Suppose that 3 balls are randomly selected from an urn containing 3 red, 4 white and 5 black balls. If we let X, Y respectively denote the number of red balls and number of white balls chosen, then find the joint probability mass function of X and Y. (4)

2) A) If X_1, X_2, X_3 are uncorrelated random variables having same standard deviation, find the correlation coefficient between $X_1 + X_2$ and $X_2 + X_3$. (3)

B) A random variable X has the p.d.f $f(x) = \frac{1}{2}e^{-|x|}$ (3)

i. Find $P(|X - \mu| \geq 2)$

ii. Use Chebyshev's inequality and verify.

C) Derive mean and variance of Gamma distribution. (4)

3) A) In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation. (3)

B) If the probability distribution function(pdf) of X is given by $f(x) = \begin{cases} 2x, & 0 \leq x \leq 1 \\ 0, & otherwise \end{cases}$, find the pdf of $Y = 3X + 1$. (3)

C) For the following data given below, find the equation to the best fitting curve of the form $y = ax^2 + bx + c$ and hence estimate y at $x = 6$ (4)

x	1	2	3	4	5
y	10	12	13	16	19

4) A) Show that $E(X-\mu)^{2n} = \sigma^{2n}\{1.3.5\ldots(2n-1)\}$ for all normal distribution with mean μ and variance σ^2. (3)

B) Let $\overline{X}$ be the mean of the random sample of size n from the distribution which has $N(\mu, 9)$. Find n such that $P(\overline{X} - 1 < \mu < \overline{X} + 1) = 0.9$. (3)

C) Find the Z transform of $a^n \cos n\theta$ and $a^n \sin n\theta$ and also using partial fractions find the inverse Z transform of $\frac{2z^2+3z}{(z+2)(z-4)}$. (4)

5) A) Prove that $\frac{d}{dx}\left(x^n J_n(x)\right) = x^n J_{n-1}(x)$. Also expand $J_1(x)$ in terms of series. (3)

B) Solve the difference equation $y_{n+2} + 3y_{n+1} + 2y_n = n$. (3)

C) Obtain the series solution of the equation $9x(1-x)\left(\frac{d^2y}{dx^2}\right) - 12\left(\frac{dy}{dx}\right) + 4y = 0$ using Frobenius method. (4)

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