Question Paper

Exam Date & Time: 18-Jul-2022 (02:00 PM - 05:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

IV B.TECH END SEMESTER MAKE-UP EXAMINATIONS, July 2022

ENGINEERING MATHEMATICS IV [MAT 2262]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions.

Instructions to Candidates: Answer ALL questions Missing data may be suitably assumed

1) A) Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, 0<x<1, 0<y<1$ subjected to $u(x,1)=u(0,y)=0,$ $u(1,y)=9(y-y^2), u(x,0)=9(x-x^2)$ by taking $h=\frac{1}{3}$. (4)

B) Solve $\frac{\partial u}{\partial t}=\frac{1}{16}\frac{\partial^2 u}{\partial x^2}, 0<x<1, t>0$ subjected to $u(x,0)=0, u(0,t)=0, u(1,t)=100t$ for one time step using crank Nicolson's scheme by taking $h=0.25$. (3)

C) Solve $xy''+y=0,\ y(1)=1,\ y(2)=2$ by taking $h=0.25$. (3)

2) A) If (X,Y) is uniformly distributed over region with vertices $(0,0),(1,0),(2,1)\ and\ (1,1)$ then find correlation coefficient between X and Y. (4)

B) The daily consumption of electric power (in million of KW-hours) is a random variable having the PDF (3)

$$f(x)=\begin{cases}\frac{1}{9}xe^{-\frac{x}{3}}, x>0\\ 0 \qquad x\le 0\end{cases}$$

If the total production is 12 million KW-hours, determine the probability that there is power cut (shortage) on any given day.

C) A dice is tossed, if the number is odd then what is the probability that it is prime . (3)

3) A) A bag contain three coins, one of which has two head and other two coins are normal. A coin is selected at random toss four time successively, if head comes up every time then what is the probability that the coin is two headed ?. (4)

B) Find the mean and variance of binomial distribution. (3)

C) Suppose the continuous random variable has joint PDF given by (3)

$$f(x,y)=\begin{cases}k(x^2+\frac{xy}{3}), 0\le x\le 1; 0\le y\le 2\\ 0, \qquad otherwise\end{cases}$$

find i) k ii) $P(X\ge\frac{1}{2})$

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4) If X and Y are two discrete random variable, having joint PDF (4)

$f(x,y)=k(2x+y), x=0,1,2; y=0,1,2,3$ ... find i) k ii) E(X) ... and E(Y)

A) $f(x,y) = k(2x+y), x = 0,1,2 ; y = 0,1,2,3$ then find i) k ii) E(X) and E(Y) .

B) The temperature X is normally distributed with mean 50 variance 4, find the probability that X lies between 48 and 53 degree. (3)

C) Using graphical method find the maximum value of $Z = 100x_1 + 40x_2$ subjected to the constraints (3)

$$\begin{aligned} 10x_1 + 4x_2 &\leq 2000 \\ 3x_1 + 2x_2 &\leq 900 \\ 6x_1 + 12x_2 &\leq 3000, \quad x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

5) Solve the following transportation problem: (4)

A)

	Destination				
	A	B	C	D	
I	21	16	25	13	11
Source II	17	18	14	23	13
III	32	27	18	41	19
Requirement	6	10	12	15	Availability

B) Consider the following problem faced by a production planner in a soft drink plant. He has two bottling machines A and B. A is designed for 8-ounce bottles and B for 16-ounce bottles. However, each can be used on both types with some loss of efficiency. The following is available. (3)

Machine	8-Ounce bottles	16-ounce bottles
A	100/minute	40/minute
B	60/minute	75/minute

The machines can be run 8 hours per day, 5 days per week. Profit in a 8-ounce bottle is 15paise and on a 16-ounce bottle is 25 paise. Weekly production of drink cannot exceed 3,00,000 ounce and the market can absorb 25,000 8-ounce bottles and 7,000 16-ounce bottles per week. The planner wishes to maximize his profit subject, of course, to all the production and marketing restriction. Formulate this as a linear programming problem.

C) Using simplex method Maximize $Z = 45x_1 + 3x_2$ subjected to the constraints (3)

$$\begin{aligned} x_1 + x_2 + &\leq 2 \\ 5x_1 + 2x_2 &\leq 10 \\ 3x_1 + 8x_2 &\leq 12, \quad x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

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