Exam Date & Time: 18-Jul-2022 (09:00 AM - 12:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

VI Semester End Semester Makeup Examination

APPLIED LINEAR ALGEBRA [MAT 5302]

Marks: 50 **Duration: 180 mins.**

Descriptive Questions

Answer all the questions. Section Duration: 180 mins

1) Show that the representation of any vectors in terms of a set of basis of vectors is unique. (3)

A)

B) In the vector space $\mathbb{R}^3$ express the vector (1, -2, 5) as a linear combination of the vectors (1, 1, 1), (1, 2, 3) and (2, -1, 1). (3)

C) Solve the following system of equations (4)

$$2x_1 - 3x_2 - 6x_3 - 5x_4 + 2x_5 = 7$$

$$x_3 + 3x_4 - 7x_5 = 6$$

$$x_4 - 2x_5 = 1.$$

2) Let W be the subspace of $\mathbb{R}^4$ generated by {(1, -2, 5, -3), (2, 3, 1,-4), (3, 8, -3, -5)}. Then find the basis of W and extend the basis of W to a basis of $\mathbb{R}^4$. (3)

A)

B) Find a linear mapping $T:\mathbb{R}^3 \to \mathbb{R}^3$ whose image is generated by (1, 0, -1) and (1, 2, 2). (3)

C) Let $F:\mathbb{R}^3 \to \mathbb{R}^3$ be a linear map defined as F(x, y, z)=(x+2y-z, y+z, x+y-2z). Then find a basis and dimension of the i). Kernel of F ii). Image of F. (4)

3) Let V be an inner product space and W is a subset of V. Then show that $W^T=\{x \in V \mid \langle x, w\rangle=0 \text{ forall } w \in W\}$ is a vector subspace of V. (3)

A)

B) Let $x, y \in \mathbb{R}^2$, where $x=(x_1, x_2)$, $y=(y_1, y_2)$, $\langle x, y\rangle=y_1(x_1+2x_2)+y_2(2x_1+5x_2)$. Is $\langle x, y\rangle$ defines an inner product on $\mathbb{R}^2$? Verify? . (3)

C) Let T: $\mathbb{R}^2 \to \mathbb{R}^4$ be a linear map defined as T(x, y)=(x, y, x+y, x-y). Then find the matrix corresponding to T relative to the ordered bases {(1, 0), (0, 1)} and {(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)} of $\mathbb{R}^2$ and $\mathbb{R}^4$ respectively. (4)

4) Using LU decomposition, solve the following equations

A)

$x_1+2x_2+4x_3=3$

$3x_1+8x_2+14x_3=13$ (3)

$2x_1+6x_2+13x_3=4$.

B) Find the minimal polynomial of $\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$. (3)

C) Find the singular value decomposition $U\Sigma V^T$ of the matrix $\begin{bmatrix} 1 & -1 \\ 1 & -1 \\ \sqrt{3} & 0 \end{bmatrix}$. (4)

5) Solve the following system of linear equations by using eigen values

$dx_1/dt=4x_1+2x_2$ and $dx_2/dt=-x_1+x_2$ with initial conditions $x_1(0)=1$ and $x_2(0)=0$. (3)

A)

B) Diagonalize the matrix $\begin{bmatrix} 2 & -2 & 1 \\ -1 & 3 & -1 \\ 2 & -4 & 3 \end{bmatrix}$, if possible. (3)

C) Apply the Gram-Schmidt orthogonalization process to find an orthonormal basis for the subspace U of R^4 spanned by (1, 1, 1, 1), (1, 2, 4, 5) and (1, -3, -4, -2). (4)

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