**END SEMESTER MAKEUP EXAMINATIONS**

**VI SEMESTER: COMPUTATIONAL LINEAR ALGEBRA JUNE 2022**

Q1. Show that S = {(x+2y, 2x – 3y)| x, y in R} is a subspace of $R^2$(3)

Q2. Let A = ($a_{ij}$) be an m×n real matrix. Define ||A|| = max |$a_{ij}$|. Show that this defines a matrix norm. Is it a consistent norm? Justify your answer.

**[A matrix norm is consistent if ||AB|| ≤||A|| ||B||]** (4)

Q3. Determine the dimension of and a basis for the row space of $A = \begin{bmatrix} 2 & -1 & 2 & 3 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & -1 \\ 1 & 1 & -3 & 4 \end{bmatrix}$(3)

Q4. Show that for any m by n matrix A with real entries $\|A\|_\infty = \max_{1 \le i \le m} \sum_{j=1}^{n} |a_{ij}|$(3)

Q5. If $F \in R^{m \times n}$ is non-singular and $||F|| < 1$, then show that $I - F$ is non-singular and that $\|(I-F)^{-1}\| \le \frac{1}{1-||F||}$ (3)

Q6. Let $A = \begin{pmatrix} 3 & 4 & 1 \\ 2 & 5 & 2 \\ 1 & -2 & 1 \end{pmatrix}$. Create a zero at (3, 1)$^{th}$ position using Givens rotation.

Take $s = \frac{1}{\sqrt{10}}, c = \frac{3}{\sqrt{10}}$ (2)

Q7. Obtain full SVD of $A = \begin{pmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{pmatrix}$ (6)

Q8. For $A = \begin{bmatrix} -2 & -1 & 1 \\ 1 & 0 & 1 \\ 3 & -1 & 4 \end{bmatrix}$ find the condition numbers $K_1(A)$ $and$ $K_\infty(A)$.

$\left[\boldsymbol{k_p(A) = \|A\|_p \|A^{-1}\|_p, \|A\|_1 = \max\{column\ sum\}, \|A\|_\infty = \max\{row\ sum\}}\right]$(3)

Q9. Suppose

$$Ax = b, A \in R^{n\times n}, 0 \neq b \in R^n, \qquad (A + \Delta A)y = b + \Delta b, \Delta A \in R^{n \times n}, \Delta b \in R^n$$

with $\|\Delta A\| \le \varepsilon \|A\|$ $and$ $\|\Delta b\| \le \varepsilon \|b\|$. If $\varepsilon k(A) = r < 1$, then show that $A + \Delta A$ is non-singular and that $\frac{\|y-x\|}{\|x\|} \le \frac{2\varepsilon k(A)}{1-r}$ (3)

Q10. Obtain LU factorization of $A = \begin{bmatrix} 7 & -2 & 1 \\ 14 & -7 & -3 \\ -7 & 11 & 18 \end{bmatrix}$ using Gaussian Elimination without pivoting and hence solve the system Ax = b where $b = \begin{pmatrix} 12 \\ 17 \\ 5 \end{pmatrix}$ (6)

Q11. Obtain LU factorization of $A = \begin{bmatrix} 1 & 3 & 4 & 0 \\ -3 & -6 & -7 & 2 \\ 3 & 3 & 0 & -4 \\ -5 & -3 & 2 & 9 \end{bmatrix}$ using Gauss Elimination with partial pivoting(4)

Q12. Obtain QR factorization of $A = \begin{bmatrix} 1 & 4 & 3 \\ 4 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ using Householder Reflections

$\left[\boldsymbol{H} = \boldsymbol{I} - \frac{2}{\boldsymbol{v^T v}}\boldsymbol{v}\boldsymbol{v^T}, \boldsymbol{v} = \boldsymbol{x} \pm \|\boldsymbol{x}\|_2 \boldsymbol{e_1}\right]$(6)

Q13. Write any two advantages of Givens rotations over Householder reflections(2)

Q14. If A in $R^{n\times n}$ is positive definite and X in $R^{n\times k}$ has rank k, then show that $B=X^TAX$ in $R^{k\times k}$ is also positive definite**.**

**[A square matrix *A* of order *n* is said to be positive definite if $x^TAx > 0$ for all non-zero vectors *x*]**(2)