END SEMESTER EXAMINATIONS

VI SEMESTER: COMPUTATIONAL LINEAR ALGEBRA MAY 2022

Type: DES

Q1. If S1 and S2 are two subspaces of a vector space V, show that $S1 \cap S2$ is also a subspace of V. Is $S1 \cup S2$ a subspace? Justify your answer. (3)

Q2. If $A \in R^{m \times n}$, then show that dim(null(A)) + rank(A) = n.

[Null(A)={x: Ax=0}, rank(A)= dim(ran(A))]. (4)

Q3. Determine the dimension of and a basis for the row space of the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 & 3 & 2 \\ 2 & -1 & 1 & 5 & 1 \\ 3 & -1 & 1 & 7 & 0 \\ 0 & 1 & -1 & -1 & -3 \end{bmatrix} (3)$$

Q4. Show that for any m by n matrix A with real entries $\|A\|_1 = \max_{1 \le j \le n} \sum_{i=1}^{m} |a_{ij}|$ (3)

Q5. If $F \in R^{m \times n}$ is non-singular and $||F|| < 1$, then show that $I - F$ is non-singular and that

$\|(I-F)^{-1}\| \le \frac{1}{1-||F||}$. (3)

Q6. Obtain the full SVD of $A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$ (6)

Q7. For $A = \begin{bmatrix} -3 & 5 & -1 \\ 4 & -1 & 2 \\ 0 & 8 & -2 \end{bmatrix}$ find the condition numbers $K_1(A)$, $K_\infty(A)$

$[\boldsymbol{k_p(A) = \|A\|_p \|A^{-1}\|_p, \|A\|_1 = \max\{column\ sum\}, \|A\|_\infty = \max\{row\ sum\}}]$ (3)

Q8. Suppose

$$Ax = b, A \in R^{n \times n}, 0 \ne b \in R^n, \qquad (A + \Delta A)y = b + \Delta b, \Delta A \in R^{n \times n}, \Delta b \in R^n$$

with $\|\Delta A\| \le \varepsilon \|A\|$ *and* $\|\Delta b\| \le \varepsilon \|b\|$. If $\varepsilon k(A) = r < 1$, then show that $A + \Delta A$ is non-singular and that $\frac{\|y\|}{\|x\|} \le \frac{1+r}{1-r}$ (3)

Q9. Obtain LU factorization of $A = \begin{bmatrix} 1 & 1 & 0 & 3 \\ 2 & 1 & -1 & 1 \\ 3 & -1 & -1 & 2 \\ -1 & 2 & 3 & -1 \end{bmatrix}$ using Gaussian Elimination

without pivoting and hence solve the system Ax = b where $b = \begin{pmatrix} 1 \\ 1 \\ -3 \\ 4 \end{pmatrix}$ (6)

Q10. Obtain LU factorization of $A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 3 & 3 & 1 \\ 8 & 7 & 9 & 5 \\ 6 & 7 & 9 & 8 \end{bmatrix}$ using Gauss Elimination with partial pivoting (4)

Q11. Obtain QR factorization of $A = \begin{bmatrix} 12 & -51 & 4 \\ 6 & 167 & -68 \\ -4 & 24 & -41 \end{bmatrix}$ using Householder Reflections $\boldsymbol{[H = I - \frac{2}{v^T v} v v^T, v = x \pm \|x\|_2 e_1]}$ (6)

Q12. Let $A = \begin{bmatrix} 1 & 5 & 5 \\ 2 & 9 & 1 \\ 6 & 8 & 8 \end{bmatrix}$. Create a zero at position (2,1) using Givens rotation.

Take $s = \frac{2}{\sqrt{5}}, c = \frac{-1}{\sqrt{5}}$. (2)

Q13. Write any two advantages of Givens rotations over Householder reflections (2)

Q14. Show that a square matrix A of order n is positive definite if and only if the symmetric matrix $T = \frac{A+A^T}{2}$ has positive eigen values.

[A square matrix A of order n is said to be positive definite if $x^T A x > 0$ for all non-zero vectors x] (2)

Commendable