Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
A Constituent Institution of Manipal University

VI SEMESTER B. TECH (OPEN ELECTIVE) END SEMESTER EXAMINATIONS, MAY- 2022

SUBJECT: MATHEMATICAL MODELLING [MAT-5304]
REVISED CREDIT SYSTEM (23/05/2022)

Time: 3 Hours **MAX. MARKS: 50**

Instructions to Candidates: Answer **All questions**.

Q No	Questions	Marks	COS
1A	Explain the twelve-point procedure for construction of new mathematical model.	**3**	**CO1**
1B	Formulate mathematical model using difference equations to determine the amount one must save monthly to build a saving account that pays an interest of 1% monthly on the amount present and allows a monthly payment of Rs 1000. Also find how much of an initial investment does one need, to deplete the annuity in 20 years	**3**	**CO3**
1C	Develop the Malthusian model for population growth. The annual birth and death rates in a country are 15.7% and 12.78% respectively, while the annual immigration and migration rates are 22.23%, 25.6%. Assuming the rates to be constant over a period of five years, use difference equation to formulate a model for population change and predict the populations of the next five years, if the current population is 685686.	4	**CO2**
2A	Define irreducible Markov chain. Prove that the Markov chain whose transition probability matrix $\begin{pmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$ is irreducible then find its corresponding stationary probability vector.	**3**	**CO4**
2B	A bacterial culture contains two strains A and B of bacteria, with respect to population of 10 million and 16 million initially. Each strain secretes a chemical that is toxic to the other, so that in an hour, each 3 bacteria of strain A kill one bacterium of strain B and each 6 bacteria of strain B kill one bacterium of strain A. Formulate a mathematical model using difference equations. Which strain will servile and how long will it take for the other to get wiped out?	**3**	**CO3**
2C	Suppose a student carrying a flu virus returns to an isolated college campus of 2000students. If it is assumed that the rate at which the virus spreads is proportional not only to the number 'x' of infected students but also to the number of students not infected, determine the number of infected students' after 6 days it is further observed that after 4 days x(4) = 100	**4**	**CO2**
3A	Two boys and two girls are throwing ball from one to other. Each boy throws the ball to the other boy with probability ½ and each girl with probability ¼. On the other hand, each girl throws the ball to each boy with probability ½ and never the other girl. Find the transition probability matrix	**3**	**CO4**

<table>
<tr><td>3B</td><td>i) Find only the equilibrium values for competitive hunter model (Spotted owls and Hawks) by using specific values for the constants $K_1 = 0.4$, $K_2 = 0.002$, $K_3 = 0.6$ and $K_4 = 0.001$
ii) With suitable assumptions Explain Harrod model, in economics and finance.</td><td>3</td><td>CO3</td></tr>
<tr><td>3C</td><td>A large vat holds 100 gallons of water that is to be mixed with sugar and then used to make soft drinks. Sugar-water containing 5 tablespoons of sugar pergallon enters the vat through a pipe at a rate of 2 gallon per minute. Another pipe pumps sugar-water with 10 tablespoons of sugar per gallon into the vat at rate of 1 gallon per minute. The vat is kept well mixed, so that the concentration of sugar in the vat is uniform. Sugar –water is drained out of the vat at rate of 3 gallon per minute. Find the amount of sugar in the vat at time t if the vat initially has 900 tablespoons in it.</td><td>4</td><td>CO2</td></tr>
<tr><td>4A</td><td>Solve the difference equation
$4\,y_{n+4} - 8\,y_{n+3} - 7\,y_{n+2} + 11\,y_{n+1} + 6\,y_n = 6000 + 2^n$</td><td>3</td><td>CO3</td></tr>
<tr><td>4B</td><td>A 20 litre tank has 20 grams of yellow food coloring dissolved in it. If a yellow food coloring solution with concentration of 2 grams/litre is piped into the tank at the rate of 3 litre/minute while the well mixed solution is drained out of the tank at the same rate, what is concentration of the yellow food coloring at time t.</td><td>3</td><td>CO3</td></tr>
<tr><td>4C</td><td>With suitable assumptions Develop a Mathematical Model for Prey- Predator model</td><td>4</td><td>CO2</td></tr>
<tr><td>5A.</td><td>A farmer has a 100 acre farm. He can sell all the tomatoes, lettuce and radishes he can raise the price of Rs 1 per kg for tomatoes, Rs, 0.75 per head for a lettuce and Rs 2 per kg for radishes the average yield per acre is 2000 kg of tomatoes, 3000 heads of lettuce and 1000 kg of radishes. Fertilizer is available at Rs 0.50 per kg and the amount required per acre is 100 kg each of tomatoes and lettuce and 50 kg for radishes. Labour required for sowing, cultivating and harvesting per acre is 5 man days for tomatoes and radishes and 6 man days for lettuce. A total of 400 man-days of labour are available at Rs 20 per man-day. Formulate an L.P. model and maximize the former total profit by using Simplex method.</td><td>3</td><td>CO5</td></tr>
<tr><td>5B.</td><td>A fertilizer company produces two products Naptha and Urea. The company gets a profit of Rs 50 per unit product of Naptha and Rs 60 per unit product of Urea. The time requirement for each product and total time available in each plant area as follows
<table>
<tr><th rowspan="2">Plant</th><th colspan="2">Hours Required</th><th rowspan="2">Available hours</th></tr>
<tr><th>Naptha</th><th>Urea</th></tr>
<tr><td>Plant A</td><td>2</td><td>3</td><td>1500</td></tr>
<tr><td>Plant B</td><td>3</td><td>2</td><td>1500</td></tr>
</table>
The demand for product is limited to 400 units. Formulate it as an LPP and solve by graphical method to find the number of units of each product to be produced in order to maximize the profit.</td><td>3</td><td>CO5</td></tr>
<tr><td>5C.</td><td>. Solve LPP using Simplex method to Maximixe $P = 6x + 2y + 3z$ subjected to
$6x + 5y + 3z \le 26, \quad 4x + 2y + 5z \le 7, \; x \ge 0, \; y \ge 0, \; z \ge 0$</td><td>4</td><td>CO5</td></tr>
</table>

$$y_{n+2} - 2\cos\alpha\, y_{n+1} + y_n = \cos n\alpha$$

$$\mu(y) = 1 - \alpha_1 y$$