Exam Date & Time: 18-Jul-2022 (09:00 AM - 12:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

VI Semester Make-Up Examination

STOCHASTIC PROCESSES AND RELIABILITY [MAT 5306]

Marks: 50 **Duration: 180 mins.**

Descriptive Questions

Answer all the questions. Section Duration: 180 mins

1) A) Find the density function of $Y = 3X + 1$ if the densiy function of X is $f(x) = 2x,$ $0 < x < 1$. (3)

B) If the random variable X is uniformly distributed in (0, 2), find the pdf of $Y = X^3$ (3)

C) Using generating function, solve the Difference equation

$$p_{n+1} - (1 + a)p_n + ap_{n-1} = 0,\ \ n \geq 1, \text{ and } -p_1 + ap_0 = 0\ \ 0 < a < 1.$$ (4)

2) A) Two random processes $X(t)$ and $Y(t)$ are define by $X(t) = A\cos w_0 t + B\sin w_0 t$ and $Y(t) = B\cos w_0 t - A\sin w_0 t$. Show that $X(t)$ and $Y(t)$ jointly wide-sense stationary, if A and B are uncorrelated random variables with zero means and same variances and w_0 is a constant. (3)

B) In the fair coin experiment, the process $\{X(t)\}$ is defined as follows

$$X(t) = \begin{cases} \sin\pi t, & \text{if head shows} \\ 2t, & \text{if tail shows.} \end{cases}$$

Find $E\{X(t)\}$ (3)

C) Show that the random process $X(t) = A\cos(\omega_0 t + \theta)$ is wide-sense stationary, if A and ω_0 are constants and θ is uniformly distributed random variable in $(0, 2\pi)$. (4)

3) A) Show that sum of two independent Poisson process is a Poisson process. (3)

B) A machine goes out of order whenever a component fails. The failure of this part follows a Poisson process with a mean rate of 1 per week. Find the probability that 2 weeks have elapsed since last failure. (3)

C) If $\{X(t)\}$ is a Poisson process, prove that

$$P\{X(s) = r \mid X(t) = n\} = \binom{n}{r}\left(\frac{s}{t}\right)^r\left(1-\frac{s}{t}\right)^{n-r}$$ where $s < t$. (4)

4) The transition probability matrix of a Markov chain $\{X_n\}$ with n=1, 2, ….. having 3 states 1, 2 and 3

A) is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $P^0 = (0.7, 0.2, 0.1)$ (3)

Find (a) $P(X_2 = 3, X_1 = 3, X_0 = 2)$

(b) $P(X_2 = 3)$

B) A college student has the study habit as follows. If he studies one night, he is 70% sure not to study the next night. If he does not study one night, he is only 60% sure not to study the next night also. Find how often he studies in the long run. (3)

C) A fair dice is tossed repeatedly. If X_n denotes the maximum of the numbers occurring in the first n tosses, find the transition probability matrix P of the Markov chain $\{X_n\}$. Also find P^2 and $P\{X_2 = 6\}$. (4)

5) A) In a single server queuing system with Poisson input and exponential service times, if the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 hour and the maximum possible number of calling units in the system is 2, find the average number of calling units in the system and in the queue. (3)

B) Given an average arrival rate of 20 per hour, is it better for a customer to get service at a single channel with mean service rate of 22 customers per hour or at one of two channels in parallel with mean service rate of 11 customers per hour for each of the two channels ? (3)

C) In a single counter supermarket, at what rate must a clerk work in order to ensure a probability of 0.90 that the customer will not wait longer than 12 min ? Assume that the customers arrive in a Poisson fashion at an average rate of 15 per hour and that the length of the service time by the clerk has an exponential distribution. (4)

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