Question Paper

Exam Date & Time: 23-May-2022 (10:00 AM - 01:00 PM)

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

STOCHASTIC PROCESSES AND RELIABILITY [MAT 5306]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions.

Instructions to Candidates: Answer ALL questions Missing data may be suitably assumed

1) Solve the Difference equation using generating function (4)

A) $u_n = qu_{n-1} + p(1-u_{n-1}), \quad n \geq 1, \ p+q=1$, with the given initial condition, $u_0 = 1$.

B) Find the density function of $Y = aX + b$ in terms of the density function of X (3)

C) Let X be a continuous random variable with pdf (3)

$$f(x) = \begin{cases} \frac{x}{12}, & in \ 1 < x < 5 \\ 0, & elsewhare \end{cases}$$

Find the probability density function of $Y = 2X - 3$

2) Show that the random process $X(t) = A\cos(\omega_0 t + \theta)$ is wide-sense stationary, if (4)

A) A and ω_0 are constants and θ is uniformly distributed random variable in $(0, 2\pi)$.

B) Find the mean and variance of the process $\{X_t; t \in T\}$whose distribution is given by (3)

$$P\{X(t) = n\} = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, if \ n = 1,2,3,\ldots \\ \frac{at}{1+at}, \ if \ n = 0 \end{cases}$$

C) In the fair coin experiment, the process $\{X(t)\}$ is defined as follows (3)

$$X(t) = \begin{cases} sin\pi t, & if \ head \ shows \\ 2t, & if \ tail \ shows. \end{cases}$$

Find $E\{X(t)\}$

3) The number of accidents in a city follows a Poisson process with mean of 2 per day the number of (4)

A) people X_i involved in the i^{th} accident has independent distribution

$P\{X_i = k\} = \frac{1}{2^k}, k \geq 1$. Find the mean and variance of the number of people involved in accidents per week.

B) If $\{N_1(t)\}$ and $\{N_2(t)\}$ are two independent Poisson processes with parameters θ_1 and θ_2 respectively. Show that (3)

$$P[N_1(t) = k | N_1(t) + N_2(t) = n] = \binom{n}{r} p^k q^{n-k}$$ where

$p = \frac{\theta_1}{\theta_1+\theta_2}$ and $q = \frac{\theta_2}{\theta_1+\theta_2}$

C) A flock of birds living in a tree return to the tree after sun set at a rate of 5 per minute in accordance with a Poisson process. If the probability of each bird arriving to the tree is 0.82, find the probability that 12 birds would arrive in a 3 minutes of interval. (3)

4) There are 2 white marbles in urn A and 3 red marbles in urn B. At each step of the process, a marble is selected from each urn and the selected marbles are interchanged. Let the stage a_i of the system be the number of red marbles in A after i changes. What is the probability that there are 2 red marbles in A after 3 steps ? (4)

A)

B) A gambler has Rs. 2. He bets Re. 1 at a time and wins Re. 1 with probability ½ . He stops playing if he loses Rs. 2 or wins Rs. 4. What is the probability that he has lost his money at the end of 5 plays ? (3)

C) A man either drives a car or catches a train to go to office each day. He never goes two days in a row by train but if he drives one day, then the next day is just as likely to drive again as he is to travel by train. On the first day of the week, he tossed a fair dice and drove to work if and only if a 6 appeared. Find the probability that he takes a train on the third day and the probability that he drives to work in the long run. (3)

5) A telephone company is planning to install telephone booths in a new airport. It has established the policy that a person should not have to wait more than 10% of the times he tries to use a phone. The demand for use is estimated to be Poisson with average of 30 per hour. The average phone call has an exponential distribution with a mean time of 5 min. How many booths should be installed ? (4)

A)

B) Given an average arrival rate of 20 per hour, is it better for a customer to get service at a single channel with mean service rate of 22 customers per hour or at one of two channels in parallel with mean service rate of 11 customers per hour for each of the two channels ? (3)

C) Customers arrive at a one-man barber shop according to a Poisson process with a mean interarrival time of 12 min. Customers spend an average of 10 min in the barber's chair. (3)

a. What is the expected number of customers in the barber shop and in the queue ?

b. Calculate the percentage of time an arrival can walk straight into the barber's chair without having to wait.

c. How much time can a customer expect to spend in the barber's shop ?

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