Question Paper

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
II SEMESTER B.Sc. (Applied Sciences) in Engg.
END SEMESTER THEORY EXAMINATION - MAY/ JUNE 2022

Mathematics - II [IMA 121]

Marks: 50   **Duration: 180 mins.**

Answer ALL questions
Missing data may be suitably assumed

1)
A) By changing the order of integration Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2 - x^2 - y^2}\, dy dx$ (3)

B) Find the volume of the portion of the sphere $x^2 + y^2 + z^2 = a^2$ lying inside the cylinder $x^2 + y^2 = ax$ (3)

C) By Using the transformation $x + y = u$, $y = uv$ Evaluate $\int_0^1 \int_0^{1-x} e^{y/x+y}\, dy dx$ (4)

2)
A) Find the directional derivative of the function $xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t$, $y = t^2$, $z = t^3$ at the point (1,1,1) (3)

B) Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$ (3)

C) Verify Greens theorem for $\oint_c 2xy\, dx - y^2\, dy$ where c s the boundary of the region bounded by the ellipse $3x^2 + 4y^2 = 12$ (4)

3)
A) Find the rank of the matrix $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ (3)

B) Test for consistency and solve by Gauss elimination method
$3x + 3y + 2z = 1$
$x + 2y = 4$
$10y + 3z = -2$
$2x \ 3y \ 5$ (3)