Question Paper

Exam Date & Time: 24-May-2022 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
II SEMESTER B.Sc. (Applied Sciences) in Engg.
END SEMESTER THEORY EXAMINATION - MAY/ JUNE 2022

Mathematics - II [IMA 121]

Marks: 50 **Duration: 180 mins.**

Answer ALL questions
Missing data may be suitably assumed

1)

A) By changing the order of integration Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2}\,dydx$ (3)

B) Find the volume bounded by portion of the sphere $x^2+y^2+z^2=a^2$ lying inside the cylinder $x^2+y^2=ax$ (3)

C) By Using the transformation $x+y=u$, $y=uv$ Evaluate $\int_0^1 \int_0^{1-x} e^{y/(x+y)}\,dydx$ (4)

2)

A) Find the directional derivative of the function $xy^2+yz^2+zx^2$ along the tangent to the curve $x=t$, $y=t^2$, $z=t^3$ at the point (1,1,1) (3)

B) Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$ (3)

C) Verify Greens theorem for $\oint_c 2xydx - y^2dy$ where c is ellipse $3x^2+4y^2=12$ (4)

3)

A) Find the rank of the matrix $A=\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ (3)

B) Test for consistency and solve by Gauss elimination method (3)

$$\begin{aligned} 3x+3y+2z&=1 \\ x+2y&=4 \\ 10y+3z&=-2 \\ 2x-3y-z&=5 \end{aligned}$$

C) Using Gauss Jordan method, find the inverse of the following matrix (4)

$$A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

4) (3)

A) Show that if a set of vectors are linearly independent then every subset is also linearly independent.

B) Test whether the following set of vectors {(1, 2, 3), (1, 1, 1), (1, 0, 1)} forms a basis for E^3. If so express (3,1,2) as linear combination of basis vectors. (3)

C) Using Gram-Schmidt process construct an orthonormal basis from the set of vectors {a1 = (1, 1, 1), a2 = (2, -1, 2), a3 = (1, 2, 3)} in E^3. (4)

5) (3)

A) *Evaluate* $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \cdot \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta$

B) A rectangular box open at the top is said to have volume of 32 cubic feet. Find the dimensions of the box requiring the least material for its construction. (3)

C) If $u = \operatorname{cosec}^{-1}\left(\frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}\right)^{1/2}$. Then prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$. (4)

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