MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

II SEMESTER M. TECH (CSE/CSIS: Program Elective I)
END SEMESTER MAKEUP EXAMINATION, 22-AUGUST-2022
SUBJECT: FUNDAMENTALS OF QUANTUM COMPUTING (CSE 5025)
REVISED CREDIT SYSTEM

Time: 3 Hours **MAX. MARKS: 50**

Note: Answer ALL the questions.

1A Let $|\psi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ and $|\psi_2\rangle = \frac{1}{\sqrt{5}}(3i|0\rangle + |1\rangle)$. Compute $\langle\psi_1 | \quad \psi_2\rangle$. 02

1B Consider the following quantum state

$$|\psi\rangle = \frac{i}{2}|00\rangle - \frac{1}{\sqrt{3}}|10\rangle + \frac{1}{2}|01\rangle + \frac{i}{\sqrt{6}}|11\rangle$$

The first qubit is measured. What is the probability that the result is 0? What is the probability that the result is 1? For each possible result, write down the post-measurement state, and calculate the probability that a measurement of the second qubit will give 0 and 1. Write down the states after the second measurement. 05

1C Imagine we can define a unitary operator U that can copy the qubit states $|\psi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$: $U|\psi_1\rangle|0\rangle = |\psi_1\rangle|\psi_1\rangle$; $U|\psi_2\rangle|0\rangle = |\psi_2\rangle|\psi_2\rangle$. Can U be used to copy $||0\rangle$ and $|1\rangle$? Verify using an explicit calculation. 03

2A In the quantum teleportation protocol, Alice and Bob are each in possession of one qubit of a pair in the joint state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. In addition, Alice has a qubit in an arbitrary state $|\theta\rangle = a|0\rangle + b|1\rangle$. Explain how the protocol works. In particular, show that it involves the transmission of exactly two classical bits of information from Alice to Bob and demonstrate how, at the end of the protocol, Bob is in possession of a qubit in state $|\theta\rangle$. 05

2B Alice wishes to exchange a private encryption key with Bob using the BB84 protocol. She generates a binary string 1010100010010111, and encode using H H H I I H I H H I I I H I I H . What is the quantum state that she transmits to Bob? Alice sends the state to Bob, who decodes using I H H I H I H H I H I I I H I H. They both reveal their choice of encoding. What is the string they retain? 03

2C Define Bell state with an example. Design quantum circuit to create Bell state. 02

3A With quantum circuit implement NAND, NOR and XOR gates using CCNOT gate. 05

3B Design the quantum circuit for TWO qubit Quantum Fourier Transform (QFT). Compute 2 qubit QFT and also give its matrix representation. 03

3C Suppose a two-qubit system is in the state $0.8|00\rangle + 0.6|11\rangle$. A Pauli X gate (i.e. a NOT gate) is applied to the second qubit, and a Hadamard gate is applied to the first qubit.

(i) What is the new state of the system? 02

(ii) What are the probabilities of the possible outcomes if both qubits are now measured

4A For the following combinations of a, N, apply Shor's algorithm to find the factors of N. 05
N = 15, a = 11

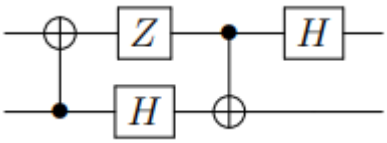
4B Show that SWAP can be implemented as a product of CNOT gates and write down the corresponding circuit. 03

4C Show that the following two qubit quantum state is an entangled state. 02

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

5A With quantum circuit design Deutsch algorithm. 05

5B Represent the following 2 qubit quantum circuit in the matrix form. 03



5C List all the stabilizer states of one qubit. 02