Question Paper

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
II SEMESTER B.Sc. (Applied Sciences) in Engg.
END SEMESTER THEORY EXAMINATION - MAY/ JUNE 2022
MATHEMATICS - II [IMA 121 - S2]

Marks: 50   **Duration: 180 mins.**

Answer all the questions.

Missing data may be suitably assumed

1)

A) By changing the order of integration Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \, dx \, dy$ (3)

B) Find the volume of the cylinder $x^2 + y^2 = 2ax$ intercepted between the paraboloid (3)

$z = \dfrac{x^2 + y^2}{2a}$ and the xy-plane.

C) Evaluate $\iint_R (x+y)^2 \, dx \, dy$ where R is the parallelogram in the xy plane with vertices (4)

(1,0) (3,1) (2,2) and (0,1) by using the transformation $u = x + y$ and $v = x - 2y$.

2)

A) Find the directional derivative of $xyz^2 + xz$ at (1,1,1) in a direction of the normal to (3)
the surface $3xy^2 + y = z$ at (0,1,1)

B) Find the constants a, b if the directional derivative of $\phi = ay^2 + 2bxy + xz$ at p(1,2,-1) (3)
is maximum in the direction of the tangent to the curve $\vec{r} = (t^2 - 1)\hat{i} + (3t - 1)\hat{j} + (t^2 - 1)\hat{k}$
at the point (0,2,0).

C) Verify Greens theorem for $\oint_c (x^2 - xy^3) dx + (y^2 - 2xy) dy$ where c is the square with (4)

vertices O(0, 0), P(2, 0), Q(2, 2) and R(0, 2).

3)   (3)

A) Find the rank of the matrix $A = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$

B) Test for consistency and solve by Gauss elimination method (3)
$x + y + z = 3$
$-x + y + z = 1$
$2x - y + 3z = 4$
$x - y - z = -1$