Question Paper

Exam Date & Time: 04-Jul-2022 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
II SEMESTER B.Sc. (Applied Sciences) in Engg.
END SEMESTER THEORY EXAMINATION - MAY/ JUNE 2022

MATHEMATICS - II [IMA 121 - S2]

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

Missing data may be suitably assumed

1) (3)

A) By changing the order of integration Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2\, dx\, dy$

B) Find the volume of the cylinder $x^2 + y^2 = 2ax$ intercepted between the paraboloid $z = \dfrac{x^2 + y^2}{2a}$ and the xy-plane. (3)

C) *Evaluate* $\iint_R (x+y)^2\, dx\, dy$ where R is the parallelogram in the xy plane with vertices (1,0) (3,1) (2,2) and (0,1) by using the transformation $u = x + y$ and $v = x - 2y$. (4)

2) (3)

A) Find the directional derivative of $xyz^2 + xz$ at (1,1,1) in a direction of the normal to the surface $3xy^2 + y = z$ at (0,1,1)

B) Find the constants a, b if the directional derivative of $\phi = ay^2 + 2bxy + xz$ at p(1,2,-1) is maximum in the direction of the tangent to the curve $\vec{r} = (t^2-1)\hat{i} + (3t-1)\hat{j} + (t^2-1)\hat{k}$ at the point (0,2,0). (3)

C) Verify Greens theorem for $\oint_c (x^2 - xy^3)dx + (y^2 - 2xy)dy$ where c is the square with vertices O(0, 0), P(2, 0), Q(2, 2) and R(0, 2). (4)

3) (3)

A) Find the rank of the matrix $A = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$

B) Test for consistency and solve by Gauss elimination method (3)

$$x + y + z = 3$$
$$-x + y + z = 1$$
$$2x - y + 3z = 4$$
$$x - y - z = -1$$

C) Using Gauss Jordan method, find the inverse of the following matrix (4)

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

4) (3)

A) Prove that a maximal linearly independent set of vectors is a basis.

B) Test whether the following set of vectors {(2, 2, 3), (-1, -2, 1), (0, 1, 0)} forms a basis for E^3. Express (3,1,2) as linear combination of basis vectors. (3)

C) Using Gram-Schmidt process construct an orthonormal basis from the set of vectors {a1 = (1, 1, 1), a2 = (-1, 0, -1), a3 = (-1, 2, 3)} in E^3. (4)

5) (3)

A) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$. Then prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \left(1 - 4\sin^2 u\right)\sin 2u$.

B) Determine the extreme values.function $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ (3)

C) Evaluate $\int_0^\infty xe^{-x^8} dx \int_0^\infty x^2 e^{-x^4} dx$ (4)

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