Exam Date & Time: 22-Jul-2022 (02:00 PM - 05:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

IV Semester End Semester Examination
Engineering Mathematics IV (MAT 2253)

ENGINEERING MATHEMATICS IV [MAT 2253]

Marks: 50 **Duration: 180 mins.**

Descriptive Questions

Answer all the questions. Section Duration: 180 mins

1) A random variable X has probability mass function $P(X = k) = \frac{c}{2^k},\ k = 0,1,2,3,4$

A) i) find the value of c, ii) find F(x) iii) find P (x is even), and iii) find $P(x \geq 5)$. (4)

B) There are 6 positive and 8 negative numbers. Four numbers are chosen at random, without replacement and multiplied. What is the probability that the product is a positive number. (3)

C) In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total. If their output 5, 4 and 2 percent are defective bolts. A bolt is drawn at random from the product and found to be defective. What is the probability that it was manufactured by machine B? (3)

2) The joint PDF of the random variable X, and Y is given by

A) $$f(x,y) = k(xy + y^2), \quad 0 \leq x \leq 1, \qquad 0 \leq y \leq 2$$

Find $P(Y > 1)$; $P\left(X > \frac{1}{2}, Y < 1\right)$ and $P(X + Y \leq 1)$. (4)

B) Fit a least square quadratic approximation for the following data. (3)

x	1	2	3	4	5	6	7
y	80	90	92	83	94	99	92

C) A target is to be destroyed in a bombing exercise. There is 75% chance that any one bomb will strike the target. Assume that two direct hits are required to destroy the target completely. How many minimum bombs must be dropped in order that the chances of destroying the target is $\geq 99\%$? (3)

3) In a distribution which is exactly normal, 12% of the items are under 30 and 85% are under 60. Find the mean and standard deviation of the distribution. (4)

A)

B) The number of arrivals of customers during any day follows Poisson distribution with mean of 5. What is the probability that the total number of customers on two days selected at random is less than 2. (3)

C) Apply graphical method to find the maximum value of $Z = 5x_1 + 3x_2$ subject to $4x_1 + 5x_2 \leq 1000; 5x_1 + 2x_2 \leq 1000; 3x_1 + 8x_2 \leq 1200; \quad x_1 \geq 0, \ x_2 \geq 0$ (3)
.

4) Use simplex method to maximize $Z = 10x + 6y + 4z$, subject to

A)

$$x + y + z \leq 100$$

$$10x + 4y + 5z \leq 600$$

$$2x + 2y + 6z \leq 300,$$

$$x, y, z \geq 0.$$

(5)

B) Consider the following linear programming problem:

$$max\, Z = x_1 + 3x_2 + 3x_3$$

subject to

$$x_1 + 2x_2 + 3x_3 = 4$$

$$2x_1 + 3x_2 + 5x_3 = 7,$$

(3)

find a) basic feasible solution, b) non-degenerate basic feasible solution, and c) optimal basic feasible solution.

C) A person wants to decide the constituents of a diet which will fulfil his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yields per unit of these foods are given in the following table:

Food Type	Yield per unit			Cost/unit (₹)
	Proteins	Fats	Carbohydrates	
1	3	2	6	55
2	4	2	4	50
3	8	7	7	95
4	6	5	4	75
Minimum requirement	1500	1000	800	

(2)

Formulate the linear programming model for the problem.

5) Find the series solution of the differential equation $8x^2y'' + 10xy' - (1 + x)y = 0.$

(4)

A)

B) Prove that $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$, where $J_n(x)$ is the Bessel's function. (3)

C) If $P_n(x)$ and $P_m(x)$ are Legendre's polynomials of degree n and m respectively, prove that

$$\int_{-1}^{1} P_m(x)P_n(x)dx = 0, \qquad n \neq m$$ (3)

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