Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

(A constituent unit of MAHE, Manipal)

FOURTH SEMESTER B.TECH (CS/IT/CC ENGINEERING) MAKE UP EXAMINATIONS, JULY- 2022

SUBJECT: ENGINEERING MATHEMATICS -IV

[MAT -2256]

REVISED CREDIT SYSTEM

(22/07/2022)

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** questions.
- All questions carry equal marks.

1A.	A bag contains three coins, one of which is two headed and the other two coins are normal and unbiased. One coin is chosen at random and is tossed four times in succession. If each time head comes up, what is the probability that this is a two headed coin?	3
1B.	The coefficients a, b, c of the quadratic equation $ax^2 + bx + c = 0$ are determined by throwing a fair die 3 times. Find the probability that (i) the roots are real. (ii) the roots are complex.	3
1C.	Suppose that (X, Y) is uniformly distributed over the triangular region $R = \{(x, y)/\ 0 \le x < y < 1\}$. Find the marginal pdf's X and Y, hence obtain the correlation coefficient.	4
2A.	A coin is known to come up heads 3 times as often as tail. This coin is tossed 3 times. Let X be the number of heads that appear. Write out the probability distribution of X and also the cumulative distribution function (c.d.f) of X.	3
2B.	(i) A bag contains 1 red and 7 white marbles. A marble is drawn from the bag and its colour is observed. Then the marble is put back into the bag and the contents are thoroughly mixed. Find the probability that in 8 such drawings, a red ball is selected exactly 3 times? (ii) Suppose that no of typographical errors on a single page of the book has a poison distribution with parameter $\alpha = 1/2$. Calculate the probability that there is at least one error on the page.	3

<table>
<tr><td>2C.</td><td>Suppose that the joint pdf of the two dimensional random variable (X, Y) is given by
$f(x,y) = \begin{cases} x^2 + \frac{xy}{3}, & 0 < x < 1,\ 0 < y < 2. \\ 0, & elsewhere. \end{cases}$
Compute $P(Y < X)$ and $P\left(Y < \frac{1}{2} \middle| X < \frac{1}{2}\right)$.</td><td>4</td></tr>
<tr><td>3A.</td><td>Let $X_1, X_2, \ldots, X_n$ denote a random sample of size n from a distribution having
p.d.f $f(x,\theta) = \begin{cases} \theta^x(1-\theta)^{1-x} & 0 \le \theta \le 1 \\ 0; & elsewhere \end{cases}$. Find a MLE for θ.</td><td>3</td></tr>
<tr><td>3B.</td><td>Derive mean and variance of a random variable which follows Exponential distribution.</td><td>3</td></tr>
<tr><td>3C.</td><td>In an examination, the marks scored by the students follow normal distribution with mean marks μ and standard deviation σ. If 10% of students got more than 90 marks and 5% have less than 35 marks, then find μ and σ.</td><td>4</td></tr>
<tr><td>4A.</td><td>If X_1, X_2 and X_3 are independent random variables and $Z = X_1 + X_2 + X_3$ and mgf of X_1, X_2 and X_3 are $(1-2t)^{-3}, (1-2t)^{-2}$ and $(1-2t)^{-\frac{1}{2}}$. Find mean and variance of $\frac{Z+1}{2}$.</td><td>3</td></tr>
<tr><td>4B.</td><td>A die is thrown 132 times with the following results
<table><tr><td>Outcome</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>Frequency</td><td>16</td><td>20</td><td>25</td><td>14</td><td>29</td><td>28</td></tr></table>Test whether the die is unbiased using chi-square test with 5% level of significance.</td><td>3</td></tr>
<tr><td>4C.</td><td>If $X \sim N(\mu, \sigma^2)$, then show that $Z = \frac{X-\mu}{\sigma} \sim N(0,1)$ and $Y = Z^2 \sim \chi^2(1)$.</td><td>4</td></tr>
<tr><td>5A.</td><td>Let the observed value of $\overline{\boldsymbol{X}}$ of size 20 from a normal distribution with mean $\boldsymbol{\mu}$ and $\boldsymbol{\sigma^2}$=80 be 81.2. Obtain 95% confidence interval for the mean $\boldsymbol{\mu}$.</td><td>3</td></tr>
<tr><td>5B.</td><td>Compute an approximate probability that the mean of a sample of size 15 from a distribution having pdf $f(x) = \begin{cases} 3x^2 & 0 < x < 1 \\ 0 & elsewhere \end{cases}$ is between 3/5 and 4/5.</td><td>3</td></tr>
<tr><td>5C.</td><td>The life length of a tyre in miles, X is normally distributed with mean θ and standard deviation 5000. The hypothesis H_0: θ ≤ 30,000 is to be tested against H_1: θ > 30,000. Observe n independent values of X, say $x_1, \ldots x_n$ and reject H_0 if and only if the sample mean $\overline{x} > c$. Find the values of n and c so that the power function $K(\theta)$ has values $K(30{,}000) = 0.01$ and $K(35{,}000) = 0.98$.</td><td>4</td></tr>
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