Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

FOURTH SEMESTER B.TECH (CS/IT/CC ENGINEERING) END SEMESTER EXAMINATIONS, JUNE 2022

SUBJECT: ENGINEERING MATHEMATICS -IV

[MAT -2256]

REVISED CREDIT SYSTEM

(14/06/2022)

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** questions.
- All questions carry equal marks.

1A.	The chances of X, Y and Z becoming managers of a certain company are in the ratio 4:2:3. The probability that bonus scheme will be introduced if X, Y and Z become managers are 0.3, 0.5 and 0.8. The bonus scheme has been introduced. What is the probability that X is appointed the manager?	**3**
1B.	Diameter of an electric cable is assumed to be a continuous random variable with probability distribution function $f(x) = \begin{cases} 6x(1-x) & 0 < x < 1 \\ 0 & otherwise \end{cases}$ i) Check it is a valid pdf ii) Obtain the expression for c.d.f iii) Compute $P\left\{X \le \frac{1}{2} \middle\vert \frac{1}{3} \le X \le \frac{2}{3}\right\}$.	**3**
1C.	A, B, C play a game and the chances of their winning it in an attempt are 2/3, 1/2 and 1/4 respectively. A has the first chance, followed by B and then by C. This cycle is repeated till one of them wins the game. Find their respective chances of winning the game.	**4**
2A.	If X, Y and Z are uncorrelated random variables with standard deviation 5, 12 and 9 respectively and $U = X + Y$, $V = Y + Z$, then find the correlation coefficient	**3**

	between U and V.	
2B.	Let $X \sim B(n, p)$ such that the sum and the product of the mean and variance of X are 24 and 128 respectively. Find the distribution completely. Hence evaluate $P(X \leq 2 \mid 1 \leq X \leq 3)$.	**3**
2C.	Suppose that the joint probability density function of X and Y is given by $f(x,y) = \begin{cases} \frac{6}{5}(x + y^2) & 0 \leq x \leq 1 \\ 0 & otherwise \end{cases}.$ Obtain the marginal probability distribution of X and Y. Also, find $P\{\frac{1}{4} \leq Y \leq \frac{3}{4}\}$.	**4**
3A.	Show that sample variance S^2 is not an unbiased estimate of population variance.	**3**
3B.	Derive mean and variance of a random variable which follows Gamma distribution.	**3**
3C.	The average marks in an entrance examination is 76 and the standard deviation is 11. i) The institution provides a special scholarship for all the students who score marks above 90. What is the percentage of students who are eligible for the scholarship? ii) It is known that 40% of the students have failed in the exam, what is the cut off for failure? iii) Calculate the percentage of failure if the cut off for failure is 70.	**4**
4A.	Find m.g.f of a random variable which is uniformly distributed over an interval $(-a, a)$, and hence find $E(X^{2n})$.	**3**
4B.	X has pdf $f(x;\theta) = \theta x^{\theta-1}, 0 < x < 1$, where $\theta > 0$. To test H_0: $\theta = 1$ against H_1: $\theta = 2$, a random sample (X_1, X_2) of size 2 is used with critical region $C = \{ (x_1, x_2) \mid 4x_1x_2 \geq 3 \}$. Compute the power function and significance level of the test.	**3**
4C.	i) If $\bar{X}$ is the mean of a random sample size n from a normal distribution with mean μ and variance 100 find n so that Pr $\{\mu - 5 < \bar{X} < \mu + 5\} = 0.954$. ii) Let $\bar{X}$ mean of a random sample of size 100 from a distribution which is j χ^2 (50) Compute an approximate value of Pr $\{49 < \bar{X} < 51\}$.	**4**
5A.	The Mendelian theory of genetics of crossing two types of peas states that the probabilities of classification of the four resulting types are 9/16, 3/16, 3/16, and 1/16 respectively. If, from 160 independent observations, the observed frequencies of these classifications are 86,	**3**

	35, 26, 13 respectively, test whether the data is consistent with the theory with level of significance α = 0.01.	
5B.	Let a random sample of size 17 from $N(\mu,\ \sigma^2)$ yields $\bar{X}$= 4.7 and S^2= 5.76. Determine 90% confidence interval for μ.	**3**
5C.	Suppose that X and Y are two independent random variables having pdf $f(x) = e^{-x}, 0 \leq x \leq \infty$ and $g(y) = 2e^{-2y}, 0 \leq y \leq \infty$. Find the pdf of X + Y.	**4**