Duration: 180 mins.

(3)

Exam Date & Time: 18-Jul-2022 (02:00 PM - 05:00 PM)



MANIPAL ACADEMY OF HIGHER EDUCATION

IV Semester End Semester Makeup Examination

ENGINEERING MATHEMATICS - IV [MAT 2258]

Marks: 50

Descriptive Questions

Answer all the questions.

1)

Solve the boundary value problem y'' - xy' + 3y = 11x with y(1) = 1.5, y(2) = 15 using the method of finite difference. Take h = 0.25. (4)

A)

B) At a telephone center, the time X (in minutes) for which an agent speaks on a telephone is found to be random variable, for which the distribution function is given by

$$f(x) = \begin{cases} kx, \ 0 \le x \le 2\\ 2k, \ 2 \le x \le 4\\ k(6-x), \ 4 \le x \le 6 \end{cases}$$

a. Find the value of k, for which f(x) is valid.

b. Find cumulative distribution function F(x).

c. Find
$$P(4 \le X < 5 \mid X > 3)$$
.

C) A bag A contains 2 white and 4 black balls. Another bag B contains 5 white and 7 black balls. A ball is transferred from bag A to bag B. Then a ball is drawn from the bag B. Find the probability that it is white.
 (3)

Solve: $\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}, \ 0 \le x \le 1, \ t > 0.$

A)
$$u(x,0) = \begin{cases} \frac{5x}{3}, & 0 \le x \le \frac{3}{5} \\ \frac{5(1-x)}{3}, & \frac{3}{5} < x \le 1, \frac{\partial u}{\partial t}(x,0) = 0 \end{cases}$$
(4)

U(0,t) = u(1,t) = 0, Take $h = \frac{1}{5}$. Compute u(x,t) for two time steps.

B) If
$$Var(X + 2Y) = 40$$
 and $Var(X - 2Y) = 20$,
a. What is $Cov(X, Y)$?
b. If $Var(X) = 2Var(Y)$, what is the correlation coefficient ρ_{XY} ?
(3)

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	C)	A group of 9 executives include 4 who are married, 3 who never married, and 2 who are divorced. Three of the executives are to be selected for promotion. Let X denotes the number of married executives and Y denotes the number of never married executives among the 3 selected for promotion. Assuming that 3 are randomly chosen from 9 available, what will be the joint probability distribution for (X,Y)?	(3)
3)		Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2$, $0 \le x \le 1.5, 0 \le y \le 1.5$,	
	A)	subject to boundary conditions $u(0, y) = u(x, 0) = 0$ and $u(x, 1.5) = u(1.5, y) = 100$. Take h=0.5.	(4)
	B)	Consider an exam that contains 10 multiple choice questions with 4 possible choices for each question, only one of which is correct. Suppose a student is to select the answer for every question randomly. Use binomial distribution to compute	
		a. the probability for the student to get no answer correct.	(3)
		b. the probability for the student to get two answers correct.	
		c. the probability that the student fails the test, i.e., to have less than 6 correct answers.	
	C)	Using Z – transform, solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$.	(3)
4)	A)	 An office has 4 secretaries handling respectively 20%, 60%, 15% and 5% of the files of a government reports. The probability that they misfile such reports are respectively 0.05, 0.1, 0.1 and 0.05. a. What is the probability that a report is misfiled? b. Find the probability that a misfiled report can be blamed on the first secretary? 	(4)
	B)	Find the inverse Z – transform of $\frac{2z^2+3z}{(z+2)(z-4)}$.	(3)
	C)	Solve: $y_{n+2} + 4y_{n+1} + 4y_n = n$.	(3)
5)	A)	Suppose that the temperature is normally distributed with mean 50 and variance 4. What is the probability that the temperature will lie between 48° C and 53° C.	(4)
	B)	Suppose the random variable X has probability density function $f(x) = \begin{cases} 2x, & 0 \le x \le 1\\ 0, & elsewhere \end{cases}$. Find the probability density function of $Y = 8X^3$.	(3)
	C)	Suppose that the random variable X has the probability density function $f(x) = \frac{1}{2}e^{- x }, -\infty < x < \infty$.Obtain the moment generating function of X and hence find its mean and variance.	(3)

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