Exam Date & Time: 09-Jun-2022 (09:00 AM - 12:00 PM)

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

ENGINEERING MATHEMATICS - IV [MAT 2258]

Marks: 50 **Duration: 180 mins.**

Descriptive Questions

Answer all the questions. Section Duration: 180 mins

1) A) Solve the boundary value problem $y'' + xy' + y = 3x^2 + 2$ with $y(0) = 0,\ y(1) = 1$ using the method of finite difference. Take $h = 0.25$. (4)

B) From a city population, the probability of selecting (i) a male or a smoker is $\frac{7}{10}$ (ii) a male smoker is $\frac{2}{5}$, and (iii) a male, if smoker is already selected is $\frac{2}{3}$. Find the probability of selecting (a) a non-smoker, (b) a male, and (c) a smoker, if male is first selected. (3)

C) Suppose that the life length of an electric device, X is considered as a continuous random variable with the following pdf $f(x) = ke^{-x}, x > 0$. The cost of manufacturing one such item is $\$2$. the manufacturer sells the item for $\$5$ but guarantees a total refund if $X \le 0.9$. What is the manufacturer's expected profit per item? (3)

2) A) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -10(x^2 + y^2 + 10), 0 < x < 3,\ 0 < y < 3$ with $u(x, y) = 0$ on the boundary and $h = \frac{1}{4}$. (4)

B) The joint pdf of two continuous random variables X and Y is given as:

$$f(x,y) = \begin{cases} \dfrac{2x+y}{210}, & 2 < x < 6, 0 < y < 5 \\ 0, & otherwise \end{cases}$$

(3)

a. Write the marginal pdfs for X and Y.

b. Find $P(X + Y > 4)$.

C) Suppose that X, Y & Z are three random variables with same variance 4. Also, the correlation coefficient between X & Y is $\frac{1}{4}$ and between Y & Z and Z & X is zero. What will be the correlation coefficient between U & V where $U = X + Y,\ V = Y + Z$? (3)

3) Compute $u(x, t)$ for four-time levels taking h= 0.25 and $\lambda = \frac{1}{2}$. Given $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1, t > 0$ subject to $u(x, 0) = 100(x - x^2)$, and $u(0, t) = u(1, t) = 0$. (4)

A)

B) If the probability that an individual will suffer a bad reaction from injection of a given serum is 0.001, determine the probability using Poisson's distribution that out of 2000 individuals

a. Exactly 3 individuals (3)

b. More than 2 individuals

will suffer a bad reaction.

C) Solve $y_{n+2} - 4y_n = n^2 + n - 1$. (3)

4)

A) Suppose that an item is produced in three factories X, Y and Z. It is known that factory X produces thrice as many items as factory Y, and that factory Y and Z produce the same number of items. If 3 percent of the items produced by each of the factories X and Z are defective, while 5 percent of those manufactured by Y are defective. All the items produced in the three factories are stocked, and an item is selected at random. (4)

a. What is the probability that the selected item is defective?

b. If an item selected is found to be defective, what is the probability that it was produced by factory Y?

B) Find the Z - transform of $u_n = \frac{1}{n!}$ and hence evaluate $Z\left(\frac{1}{(n+1)!}\right)$. (3)

C) Using Z – transform, solve $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$, with $u_0 = 0, u_1 = 1$. (3)

5)

A) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and variance of distribution. (4)

B) Suppose X is uniformly distributed over $(0,1)$, find pdf of $Y = \frac{1}{X+1}$. (3)

C) Suppose the moment generating functions of three independent random variables X_1, X_2 and X_3 are $e^{2t(1+t)}$, $e^{3t(1+t)}$ and $e^{4t(1+t)}$ respectively. Then find the probability density function of (3)

$Z = 4X_1 + X_2 + 2X_3$. Hence obtain $E\left(\frac{Z}{2}\right)$ and $V\left(\frac{Z}{2}\right)$.

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