Question Paper

Exam Date & Time: 25-Jul-2022 (02:00 PM - 05:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

IV SEMESTER B. TECH END SEMESTER MAKE-UP EXAMINATIONS, JULY 2022

ENGINEERING MATHEMATICS IV [MAT 2255]

Marks: 50 **Duration: 180 mins.**

DESCRIPTIVE TYPE

Answer all the questions. Section Duration: 180 mins

1)

A) Solve $xy'' + y = 0$ subject to the conditions $y(1) = 1,\ y(2) = 2$ by taking $h = 0.25$ by finite difference method. (4)

B) Let $\bar{X}$ be the mean of a random sample of size 100 from a distribution which is $\chi^2(50)$. Compute an approximate value of $P(49 < \bar{X} < 51)$ using central limit theorem. (3)

C) Find the extremum of the functional $\int_{x_0}^{x_1}\{y' + x^2(y')^2\}dx$ (3)

2)

A) Let $\bar{X}$ be the random sample of size 6 from a normal distribution $N(0,125)$. Find C, so that $P\{\bar{X} < C\} = 0.9$. Also, find $P\{33.542 < S^2 < 55.625\}$. (4)

B) Suppose that X is a random variable with pdf given by $f(x) = 2x, 0 \leq x \leq 1$. Find the pdf of $Y = 8X^3$. (3)

C) Suppose that X is uniformly distributed over (-a, a) where a > 0. Whenever possible determine 'a' so that the following conditions are satisfied. (3)

a) $P(x > 1) = \frac{1}{3}$ b) $P\left(x < \frac{1}{2}\right) = 0.7$

3)

A) Solve using simplex method. (4)

Maximize $Z = 5x_1 + 3x_2$ subject to $x_1 + x_2 \leq 2$, $5x_1 + 2x_2 \leq 10$, $3x_1 + 8x_2 \leq 12$, $x_1, x_2 \geq 0$

B) Derive the mean and variance of exponential distribution. (3)

C) .Find the curves on which the functional $\int_0^1((y')^2 + 12xy)dx$ with y(0)=0 and y(1)=1 can be extremized. (3)

4) (4)

A) Show that for the normal distribution with mean μ and variance σ^2, $E[(X-\mu)^{2n}] = 1.3.5\ldots(2n-1)\sigma^{2n}$.

B) Solve using Graphical method. (3)
Maximize $Z = 8x_1 + 5x_2$ subject to $2x_1 + x_2 \leq 500$, $x_1 \leq 50$, $x_2 \leq 250$ $x_1, x_2 \geq 0$.

C) Let X and Y be two independent random variables with pdf's (3)
$f(x) = e^{-x}, x > 0, g(y) = 2e^{-2y}, y > 0$. Find the pdf of the random variable $Z = X + Y$.

5) (4)

A) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, t > 0$ with the boundary conditions $u(0,t) = 0, u(4,t) = 0, u(x,0) = \frac{x}{3}(16 - x^2)$. Obtain $u_{i,j}$ for one time step with $h = 1, \lambda = 1$, using Crank-Nicolson method.

B) In a consignment of electric lamps 5% are defective. If a random sample of 8 lamps are inspected, what is the probability that one or more lamps are defective. (3)

C) Solve the transportation problem. (3)

	Destination					
	A	B	C	D		
Source I	11	13	17	14	250	Availability
Source II	16	18	14	10	300	
Source III	21	24	13	10	400	
Requirement	200	225	275	250		

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