IV SEMESTER B. TECH. (MECHATRONICS)

END SEMESTER EXAMINATIONS JUNE 2022

SUBJECT: ENGINEERING MATHEMATICS-IV [MAT-2261]

Time: 3 Hours

Date: 11-06-2022

MAX MARKS: 50

Instructions to Candidates

Answer all the questions (3+3+4)

Missing data may be suitable assumed and statistical tables may be used.

- **1 A**. The chance that a doctor A will diagnose the disease correctly is 60%. The chance that the patient of A will die after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. If a patient of A dies. What is the probability that his disease was diagnosed correctly?
 - B. (i) A student takes his examination in four subjects P, Q, R and S. He estimates his chances of passing in P as ⁴/₅, in Q as ³/₄, in R as ⁵/₆, in S as ²/₃. To qualify he must pass in P and at least two other subjects. What is the probability that he qualifies?
 (ii) If μ = 10 and σ² = 4. Find the bound for P(5 < X < 15).
 - C. Suppose that a 2-dimensional continuous random variable has the joint p. d. f.

$$f(x,y) = \begin{cases} kx(x-y); & |y| < x, & 0 < x < 2\\ 0; & otherwise \end{cases}$$
. (i) Find k

(ii)Marginal p. d. f. of Y. (iii) Find the conditional p. d. f. of X given Y.

- **2 A.** If *X*, *Y*, and *Z* are uncorrelated random variable with standard deviation 5, 12, 9 respectively. If U = X + Y and V = Y + Z. Evaluate the correlation coefficient between *U* and *V*.
 - **B.** Find the moment generating function (MGF) for Gamma distribution $X \sim G(r, \alpha)$. Also obtained mean and variance of Gamma distribution using MGF.
 - **C.** Let X be a random variable taking values $\{-2, -1, 0, 1, 2\}$, such that

$$P(X = -2) = P(X = -1), P(X = 2) = P(X = 1)$$
 and

P(X > 0) = P(X < 0) = P(X = 0). Obtain the probability mass function of X and its cumulative distribution function.

3 A. In a normal distribution 31% of the items are under 45 and 48% are over 64. Find the mean and variance of the distribution.

B. Let X be a random variable with probability distribution $f(x) = \frac{1}{\pi(1+x^2)}$ for

 $-\infty < x < \infty$. Then find the pdf of $Y = \frac{1}{x}$.

C. Fit a curve of the form $y = ab^x$ for the data and hence find the estimation for y, when x = 8.

Х	1	2	3	4	5	6	7
У	87	97	113	129	202	195	193

- **4 A.** Solve the difference equation $y_{n+2} 4y_n = n(n-1)$.
 - **B.** Find the Z-transform of (i) $n \sin n\theta$ (ii) $\sin(3n + 5)$.
 - **C.** Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using Z-transform.
- **5** A. Prove that $\frac{d}{dx} \{x^{-n}J_n(x)\} = -x^{-n}J_{n+1}(x)$. Also expand $J_1(x)$ in terms of series.
 - **B.** If \overline{X} is the mean of a random sample from a normal distribution with mean μ and variance 100. Find n such that $P(\mu 5 < \overline{X} < \mu + 5) = 0.954$.
 - **C.** Obtain the series solution of the equation 4xy'' + 2y' + y = 0 using Frobenius method.