

Reg. No.									
----------	--	--	--	--	--	--	--	--	--

MANIPAL ACADEMY OF HIGHER EDUCATION

DEPARTMENT OF SCIENCES

FOURTH SEMESTER MSc: END SEMESTER EXAMINATION (MAY 2022)

SUBJECT: GENERAL RELATIVITY AND COSMOLOGY (PHY 6202)

(CREDIT SYSTEM)

TIME: 3 HOURS

MAX. MARKS: 50

Note: Missing data may be suitably assumed

$$G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg/s}^2$$

$$M_{\odot} = 2 \times 10^{30} \text{ kg}$$

$$R_{\odot} = 7 \times 10^8 \text{ m}$$

$$c = 3 \times 10^8 \text{ m/s}$$

$$1\text{pc} = 3.08 \times 10^{16} \text{ m}$$

Curvature tensor:  $R^{\kappa}_{\lambda\mu\nu} = \partial_{\mu}\Gamma^{\kappa}_{\lambda\nu} - \partial_{\nu}\Gamma^{\kappa}_{\lambda\mu} + \Gamma^{\kappa}_{\mu\alpha}\Gamma^{\alpha}_{\lambda\nu} - \Gamma^{\kappa}_{\nu\alpha}\Gamma^{\alpha}_{\lambda\mu}$

Covariant derivative of covariant vector:  $A_{\mu;\alpha} = A_{\mu,\alpha} - \Gamma^{\beta}_{\mu\alpha}A_{\beta}$

- 1A Derive Lorentz transformation equation for space-time coordinates. 5M
- 1B An astronaut must journey to a distant planet, which is 200 light-years from Earth. What speed will be necessary if the astronaut wishes to age only 10 years during the round trip? 3M
- 1C Discuss parallel displacement of vector  $A^{\mu}$  in curved space (covariant differentiation). 2M
- 2A Show that four-velocity and four-acceleration are orthogonal. 5M
- 2B A muon has a lifetime of  $2 \times 10^{-6}$  s in its rest frame. It is created 100 km above the earth and moves towards it at a speed of  $2.97 \times 10^8$  m/s. At what altitude does it decay? According to the muon, how far did it travel in its brief life? 3M
- 2C Show that, in non-relativistic system, expression for kinetic energy reduces to  $mv^2/2$ . 2M

- 3A Obtain Riemann tensor for a curved surface. 5M
- 3B Obtain an expression for Einstein angle and mention its significance. 3M
- 3C What is the significance of geodesic coordinate system? 2M
- 4A Obtain Schwarzschild solution for Einstein's equation inside a spherically symmetric gravitating matter made up of perfect fluid. 5M
- 4B Find the gravitational redshift for radiation emitted from the surface of the Sun and for radiation emitted from a white dwarf, whose radius is 1% that of the Sun. 3M
- 4C Suppose we observe the image of a quasar to be shifted by 5 arc sec on the sky. What is the mass of the intervening galaxy? Assume that the light passes past the edge of the galaxy's disk, which has a radius of 50 kpc.
- Note: Express mass in solar mass units. 2M
- 5A Obtain Einstein's field equation from weak gravitational field approximation. 5M
- 5B Calculate  $\frac{\partial}{\partial x_k} (a_{ij} x_i (x_j)^2)$  where  $a_{ij} = a_{ji}$  are constants. 3M
- 5C Calculate Schwarzschild radius for an object of 2 solar mass. 2M

\*\*\*\*\*