Exam Date & Time: 23-May-2022 (10:00 AM - 01:00 PM)

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

VI Semester End Semester Examination-Applied Linear Algebra (MAT 5302)

**APPLIED LINEAR ALGEBRA [MAT 5302]**

**Marks: 50** **Duration: 180 mins.**

**Descriptive Questions**

**Answer all the questions.** Section Duration: 180 mins

1) A) Let V be a finite dimensional vector space. Then prove that any two bases of V have the same number of elements. (3)

B) In the vector space $R^4$ determine whether or not the vector (3, 9, -4, -2) is a linear combination of the vectors (1, -2, 0, 3), (2, 3, 0, -1) and (2, -1, 2, 1)? (3)

C) Solve the following system of equations (4)

$$x_1 - 3x_2 + 2x_3 - x_4 + 2x_5 = 2$$

$$3x_1 - 9x_2 + 7x_3 - x_4 + 3x_5 = 7$$

$$2x_1 - 6x_2 + 7x_3 + 4x_4 - 5x_5 = 7.$$

2) A) Let W be the subspace of $R^5$ generated by {(1, 0, 1, 0, 1), (1, 1, 2, 1, 0), (2, 1, 3, 1, 1), (1, 2, 1, 1, 1)}. Then find the basis of W and extend the basis of W to a basis of $R^5$. (3)

B) Let V and W be vector spaces and $\{V_1, V_2, \ldots, V_n\}$ be a basis of V. Let $W_i$, $1 \le i \le n$ be any set of(not necessarily distinct) vectors in W. Then show that there exists a unique linear map $T:V \to W$ such that $T(v_i)=w_i$. (3)

C) Let $F:R^3 \to R^3$ be a linear map defined as F(x, y, z)=(x+2y-3z, 2x+5y-4z, x+4y+z). Then find a basis and dimension of i). Kernel of F ii). Image of F (4)

3) A) Let W be the subspace of $R^4$ spanned by the vectors {(1, 1, 0, 2), (2, 0, 1, 1), (1, -1, 1, -1)}. Find a basis for orthogonal complement of W. (3)

B) Define an inner product on a vector space. Let $f, g \in C[0,1]$ (where $C[0,1]$ is the vector space of all continuous functions on [0, 1], that is, where $0 \le t \le 1$) and (3) define $< f, g >= \int_0^1 f(t)g(t)dt$ . Does $< f, g >$ forms an inner product on $C[0,1]$ ? Verify?

C) Let V=$R^3$ and T:V→V be a linear map defined by T(x, y, z)=(x+z, -2x+y, -x+2y+z). Then find the matrix corresponding to T relative to the ordered basis {(1, 0,1), (-1, 1, 1), (0, 1, 1)}. (4)

4)

A) Find the minimal polynomial of $\begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ (3)

B) Using LU decomposition, solve the following equations

$7x_1 - 2x_2 + x_3 = 12$

$14x_1 - 7x_2 - 3x_3 = 17$ (3)

$-7x_1 + 11x_2 + 18x_3 = 5$

C) Consider the subspace U of $R^4$ spanned by the vectors: {(1, 1, 1, 1), (1, 1, 2, 4), (1, 2, -4, -3)}. By using Gram-Schmidt process construct an orthonormal basis of U. (4)

5)

Diagonalize the matrix $\begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$, if possible. (3)

A)

B) Solve the following system of linear equations by using eigen values.

$dx_1/dt=4x_1+2x_2$ and $dx_2/dt= -x_1+x_2$ with initial conditions $x_1(0)=1$ and $x_2(0)=0$. (3)

C) Find the singular value decomposition $U\Sigma V^T$ of the matrix

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$ (4)

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