## Exam Date & Time: 19-May-2022 (10:00 AM - 01:00 PM)



VI Semester Program Elective (Common to all Branches) End Semester Examination, 19th MAY 2022

## APPLIED STATISTICS AND TIME SERIES ANALYSIS [MAT 4051]

Marks: 50

Answer all the questions.

## **Descriptive Questions**

Section Duration: 180 mins

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If particles are emitted from a radio-active source at the rate of 20 per hour, find the probability that (i) exactly 5 particles are emitted during a 15 minute period & (ii) fewer than 3 particles are emitted during a 12 minute period. Assume that the counting process is Poisson with probability mass function (3)

$$P[N(t) = n] = \frac{e^{-\lambda t} (\lambda t)^n}{n!}, n = 0, 1, 2, ..., \lambda > 0$$

- B) Television sets for repair arrive at random at an average rate of 4 per day to a single repairman who takes an average of  $1^{1/2}$  hours to carry out each repair, it being assumed that the repair times have a negative exponential distribution. What is the average number of television sets in the workshop? The repairman works an eight hour day (3)
- C) Obtain the trend eliminated values using additive model on fitting a straight line trend of the form  $U_t = a + bt$  to the following data using the method of least squares. Also estimate the trend for 2025.

Year	Sales in lakhs
t	Ut
1961	25
1971	25.1
1981	27.9
1991	31.9
2001	36.1
2011	43.9

(4)

**2021** 54.7

2)

Consider a stochastic process X(t) whose mean function is m(t) = 3 and autocorrelation function is  $R(t_1, t_2) = 9 + 4e^{-0.2|t_1 - t_2|}$ . Find the mean, variance and the covariance of the random variables Z = X(5) and W = X(8) (3)

B) Examine the process 
$$\left\{X(t) = \cos(t+\emptyset), \emptyset \text{ is uniform in } \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right\}$$
 for stationarity. (3)

C) Compute the potential matrix corresponding to the given transition probability matrix P of a Markov chain having state space {1,2,3} and hence obtain the probabilities of ever reaching state j from state i for all i, j.

	[0.3	0.7	٥٦	
P =	0.4	0.4	0.2	
	lο	0	1	

3)

4)

A)

Briefly explain the types of stochastic processes with one example each.

(3)

A)

- B) In a small barber shop, only one customer can get hair cut while another customer can wait in a chair. Any other arriving customer has to wait outside as there is only one chair available. The customers arrive randomly at 6 per hour. The service is exponential and takes 6 minutes on the average. Find the probability that an arriving (3) customer will have to wait outside?
- C) Suppose that the weather on any day depends on the weather conditions for the previous days, i.e., suppose it was sunny today and yesterday, then it will be sunny tomorrow with probability 0.8; if it was sunny today but cloudy yesterday, then it will be sunny tomorrow with probability 0.6; if it was cloudy today but sunny yesterday, then it will be sunny tomorrow with probability 0.4; if it was cloudy for the last two days, then it will be sunny tomorrow with probability 0.1. Transform this (4) model into a Markov chain and write its transition probability matrix. Find the stationary distribution of the Markov chain. On what fraction of the days in the long run, is it sunny?
- For the following series of observations, obtain the 5-yearly weighted moving average with weights 1, 2, 2, 2, 1 respectively.
  - 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020 2021 Year (3)Annual sales (Rs. 2 6 5 3 7 4 8 1 2 6 3 (000)

- B) The arrival pattern of cars to the local oil change centre follows a Poisson distribution at a rate of 4 per hour. If the time to perform an oil change is exponentially distributed and requires on average of 12 minutes to carry out, determine the probability of finding more than 3 cars waiting for the single available (3) mechanic to service their car?
- C) Lorries arrive at an average rate of 8 per day at an unloading bay which operates 24 hours a day. The time to unload a lorry averages 2 hours. It is assumed that inter arrival times and unloading times have negative exponential distributions. Find the average number of lorries in the yard, waiting to be unloaded but excluding the one (4) at the bay. If this number exceeds 3, serious congestion problems can arise. Is this likely to happen?
- 5) Use the method of simple averages to determine the monthly indices for the following data regarding production (in lakhs of tonnes) of a commodity for the following years.

A)

Month	Year		
2019	2020	2021	
January	12	15	16
February	11	14	15
March	10	13	14
April	14	16	16
May	15	16	15
June	15	15	17
July	16	17	16
August	13	12	13
September	11	13	10
October	10	12	10
November	12	13	11

(3)

December	15	14	15	
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B) Explain briefly, the four distinct components of a time series

(3)

(4)

C) For the transition probability matrix given below having state space {1,2,3,4,5,6,7}, determine the transient, recurrent and absorbing states if any, using transition diagram. If there exists recurrent states, identify whether they are null recurrent or non-null recurrent, & if non-null recurrent, determine their period.

0	0	1	0	0	0	0
0	0.2	0	0	0.4	0.4	0
0.8	0	0	0	0.2	0	0
0	0	0	0	0	1	0
0	0	1	0	0	0	0
0	0	0	0.7	0	0.3	0
0	0	0	0	0	0	1

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