MAT 5305

Exam Date & Time: 23-May-2022 (10:00 AM - 01:00 PM)



MANIPAL ACADEMY OF HIGHER EDUCATION

IV Semester End Semester Examination Engineering Mathematics IV (MAT 2256)

OPTIMIZATION TECHNIQUES [MAT 5305]

Marks: 50

Answer all the questions.

Descriptive Questions

Section Duration: 180 mins

Duration: 180 mins.

1) Define solution, basic feasible solution and optimal solution of an LPP

(3)

(5)

A)

A)

- B) Express the following problem in standard form: *Minimize* $Z = 3x_1 + 4x_2$ subject to $2x_1 - x_2 - 3x_3 = -4$; $3x_1 + 5x_2 + x_4 = 10$; $x_1 - 4x_2 = 12$, $x_1, x_3, x_4 \ge 0$ (3)
- C) A company produces two types of cow boy hats. Each hat of the first type requires twice as much labour time as the second type. If all hats are of the second type only, the company can produce a total of 500 hats a day. The market limits the daily sales of the first and second types to 150 and 250 hats. Assuming that the profit per hat are Rs.8 for type I and Rs.5 for type II, formulate the problem as an LPP in order to maximize the profit.
- 2) Solve by Simplex method: $Minimize Z = x_1 3x_2 + 2x_3$

subject to $3x_1 - x_2 + 3x_3 \le 7$;

$$-2x_1 + 4x_2 \le 12; \quad -4x_1 + 3x_2 + 8x_3 \le 10, \qquad x_1, x_2, x_3 \ge 0$$

B) Solve by Two-Phase method: $Min Z = 6x_1 + 5x_2$ subject to $2x_1 + x_2 \ge 80$; $x_1 + 2x_2 \ge 60$; $x_1, x_2 \ge 0$ (5)

A steel company has three open hearth furnaces and five rolling mills. (5)
Transportation cost (Rupees per quintal) for shipping steel from furnaces to rolling mills are given in the following table:



A)

MAT 5305

| FurnaceMill | M1 | M2 | M3 | M4 | Capacity |
|-------------|----|----|----|----|----------|
| P1 | 6 | 1 | 9 | 3 | 70 |
| Р2 | 11 | 5 | 2 | 8 | 55 |
| Р3 | 10 | 12 | 4 | 7 | 70 |
| Requirement | 85 | 35 | 50 | 45 | |

How should the steel be transported in order that the transportation cost is minimum? What is the minimum cost?

- B) Define Predecessor activity, successor activity and dummy activity. Give one example
- C) Define optimum strategy and mixed strategy in game theory
- 4) Solve the following assignment problem for minimizing the cost:

| | Ι | II | III | IV |
|---|----|----|-----|----|
| A | 32 | 26 | 35 | 38 |
| B | 27 | 24 | 26 | 32 |
| С | 28 | 22 | 25 | 34 |
| D | 10 | 10 | 16 | 16 |

B) A project has the following time schedule:

| Activity | 1-2 | 1-3 | 1-4 | 2-5 | 3-6 | 3-7 | 4-6 | 5-8 | 6-9 | 7-8 | 8-9 |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Duration | 2 | 2 | 1 | 4 | 8 | 5 | 3 | 1 | 5 | 4 | 3 |

(5)

(5)

(3)

(2)

MAT 5305

Construct the network and compute (i)EST, LST, EFT, LFT of the activities (ii) Critical path and its duration

5) The following table lists the jobs of a network along with their time estimates: (5)

,

A)

| Activity | t _o | t _m | t _p |
|----------|----------------|----------------|----------------|
| 1-2 | 3 | 6 | 15 |
| 1-6 | 2 | 5 | 14 |
| 2-3 | 6 | 12 | 30 |
| 2-4 | 2 | 5 | 8 |
| 3-5 | 5 | 11 | 17 |
| 4-5 | 3 | 6 | 15 |
| 6-7 | 3 | 9 | 27 |
| 5-8 | 1 | 4 | 7 |
| 7-8 | 4 | 19 | 28 |

(i)Draw the project network, calculate the length and variance of the critical path.(ii) What is the probability that the project will not be completed within 45 days (iii) Find the due date which has 95% chance to meet

MAT 5305

| | STANDAL | RD NORM | AAL DIST | RIBUTI | ON: Table | Values R | epresent / | REA to t | he LEFT | of the Z so | ore. |
|--|---------|---------|----------|--------|-----------|----------|------------|----------|---------|-------------|--------|
| | Z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
| | 0.0 | .50000 | .50399 | .50798 | .51197 | .51595 | .51994 | .52392 | .52790 | .53188 | .53586 |
| | 0.1 | .53983 | .54380 | .54776 | .55172 | .55567 | .55962 | .56356 | .56749 | .57142 | .57535 |
| and the second s | 0.2 | .57926 | .58317 | .58706 | .59095 | .59483 | .59871 | .60257 | .60642 | .61026 | .61409 |
| | 0.3 | .61791 | .62172 | .62552 | .62930 | .63307 | .63683 | .64058 | .64431 | .64803 | .65173 |
| | 0.4 | .65542 | .65910 | .66276 | .66640 | .67003 | .67364 | .67724 | .68082 | .68439 | .68793 |
| | 0.5 | .69146 | .69497 | .69847 | .70194 | .70540 | .70884 | .71226 | .71566 | .71904 | .72240 |
| | 0.6 | .72575 | .72907 | .73237 | .73565 | .73891 | .74215 | .74537 | .74857 | .75175 | .75490 |
| | 0.7 | .75804 | .76115 | .76424 | .76730 | .77035 | .77337 | .77637 | .77935 | .78230 | .78524 |
| | 0.8 | .78814 | .79103 | .79389 | .79673 | .79955 | .80234 | .80511 | .80785 | .81057 | .81327 |
| | 0.9 | .81594 | .81859 | .82121 | .82381 | .82639 | .82894 | .83147 | .83398 | .83646 | .83891 |
| | 1.0 | .84134 | .84375 | .84614 | .84849 | .85083 | .85314 | .85543 | .85769 | .85993 | .86214 |
| | 1.1 | .86433 | .86650 | .86864 | .87076 | .87286 | .87493 | .87698 | .87900 | .88100 | .88298 |
| | 1.2 | .88493 | .88686 | .88877 | .89065 | .89251 | .89435 | .89617 | .89796 | .89973 | .90147 |
| | 1.3 | .90320 | .90490 | .90658 | .90824 | .90988 | .91149 | .91309 | .91466 | .91621 | .91774 |
| | 1.4 | .91924 | .92073 | .92220 | .92364 | .92507 | .92647 | .92785 | .92922 | .93056 | .93189 |
| | 1.5 | .93319 | .93448 | .93574 | .93699 | .93822 | .93943 | .94062 | .94179 | .94295 | .94408 |
| | 1.6 | .94520 | .94630 | .94738 | .94845 | .94950 | .95053 | .95154 | .95254 | .95352 | .95449 |
| | 1.7 | .95543 | .95637 | .95728 | .95818 | .95907 | .95994 | .96080 | .96164 | .96246 | .96327 |
| | 1.8 | .96407 | .96485 | .96562 | .96638 | .96712 | .96784 | .96856 | .96926 | .96995 | .97062 |
| | 1.9 | .97128 | .97193 | .97257 | .97320 | .97381 | .97441 | .97500 | .97558 | .97615 | .97670 |
| | 2.0 | .97725 | .97778 | .97831 | .97882 | .97932 | .97982 | .98030 | .98077 | .98124 | .98169 |
| | 2.1 | .98214 | .98257 | .98300 | .98341 | .98382 | .98422 | .98461 | .98500 | .98537 | .98574 |
| | 2.2 | .98610 | .98645 | .98679 | .98713 | .98745 | .98778 | .98809 | .98840 | .98870 | .98899 |
| | 2.3 | .98928 | .98956 | .98983 | .99010 | .99036 | .99061 | .99086 | .99111 | .99134 | .99158 |
| | 2.4 | .99180 | .99202 | .99224 | .99245 | .99266 | .99286 | .99305 | .99324 | .99343 | .99361 |
| | 2.5 | .99379 | .99396 | .99413 | .99430 | .99446 | .99461 | .99477 | .99492 | .99506 | .99520 |
| | 2.6 | .99534 | .99547 | .99560 | .99573 | .99585 | .99598 | .99609 | .99621 | .99632 | .99643 |
| | 2.7 | .99653 | .99664 | .99674 | .99683 | .99693 | .99702 | .99711 | .99720 | .99728 | .99736 |
| | 2.8 | .99744 | .99752 | .99760 | .99767 | .99774 | .99781 | .99788 | .99795 | .99801 | .99807 |
| | 2.9 | .99813 | .99819 | .99825 | .99831 | .99836 | .99841 | .99846 | .99851 | .99856 | .99861 |
| | 3.0 | .99865 | .99869 | .99874 | .99878 | .99882 | .99886 | .99889 | .99893 | .99896 | .99900 |
| | 3.1 | .99903 | .99906 | .99910 | .99913 | .99916 | .99918 | .99921 | .99924 | .99926 | .99929 |
| | 3.2 | .99931 | .99934 | .99936 | .99938 | .99940 | .99942 | .99944 | .99946 | .99948 | .99950 |
| | 3.3 | .99952 | .99953 | .99955 | .99957 | .99958 | .99960 | .99961 | .99962 | .99964 | .99965 |
| | 3.4 | .99966 | .99968 | .99969 | .99970 | .99971 | .99972 | .99973 | .99974 | .99975 | .99976 |
| | 3.5 | .99977 | .99978 | .99978 | .99979 | .99980 | .99981 | .99981 | .99982 | .99983 | .99983 |
| | 3.6 | .99984 | .99985 | .99985 | .99986 | .99986 | .99987 | .99987 | .99988 | .99988 | .99989 |
| | 3.7 | .99989 | .99990 | .99990 | .99990 | .99991 | .99991 | .99992 | .99992 | .99992 | .99992 |
| | 3.8 | .99993 | .99993 | .99993 | .99994 | .99994 | .99994 | .99994 | .99995 | .99995 | .99995 |
| | 3.9 | .99995 | .99995 | .99996 | .99996 | .99996 | .99996 | .99996 | .99996 | .99997 | .99997 |

B) Solve the following game:

| | B1 | В2 | В3 | В4 | В5 | B6 |
|----|----|----|----|----|----|----|
| A1 | 4 | 2 | 0 | 2 | 1 | 1 |
| A2 | 4 | 3 | 1 | 3 | 2 | 2 |
| A3 | 4 | 3 | 7 | -5 | 1 | 2 |
| A4 | 4 | 3 | 4 | -1 | 2 | 2 |
| A5 | 4 | 3 | 3 | -2 | 2 | 2 |

(5)

-----End-----