Question Paper

Exam Date & Time: 14-Jan-2023 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

FIRST SEMESTER B.TECH. EXAMINATIONS - JANUARY 2023
SUBJECT: MAT 1171 / MAT-1171 - ENGINEERING MATHEMATICS - I

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

1A) A survey conducted in a slum locality reveals the following information as classified below: (4)

Income per day (Rs.)	Under 10	10-20	20-30	30-40	40-50
Number of persons	20	45	115	210	115

Estimate the probable number of persons in the income group 20-25.

1B) Solve $\frac{dy}{dx} = \frac{(y^4+2y)}{4x-xy^3-2y^4}$ (3)

1C) From the data given below, Compute $\left(\frac{dy}{dx}\right)_{x=3}$ and $\left(\frac{d^2y}{dx^2}\right)_{x=-2}$. (3)

x	-2	-1	0	1	2	3
y	0	1	6	24	60	120

2A) Using Runge Kutta method of fourth order, solve $y' = x + y^2$ with $y(0) = 1$ at $x = 0.2$ in steps of length $h = 0.1$. (4)

2B) Solve $(3y^4 + 3x^2y^2)dx + (x^3y - 3xy^3)dy = 0$. (3)

2C) Find the approximate value of $\int_0^{\frac{\pi}{2}} \sqrt{\cos\theta}\ d\theta$ using Simpson's 1/3rd rule by dividing $\left[0, \frac{\pi}{2}\right]$ into 6 equal parts. (3)

3A) Using Gauss Seidel method solve the system of equations (4)

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$
$$-2x_1 + 10x_2 - x_3 - x_4 = 15$$
$$-x_1 - x_2 + 10x_3 - 2x_4 = 27$$
$$-x_1 - x_2 - 2x_3 + 10x_4 = -9$$

Take the initial approximation as $x_2 = x_3 = x_4 = 0$.
Carry out 3 iterations and correct up to 4 decimal places.

3B) Solve the differential equation $(D^2 + 1)y = \sec x\ \tan x$ by the method of variation of parameters. (3)

3C) Compute a real root of $3x - \cos x - 1 = 0$ using the Newton-Raphson method with $x_0 = 0.6$ correct to four decimal places. (3)

4A) Using Gram-Schmidt orthogonalization process, construct an orthonormal set of vectors from the set $\{(1,2,1),\ (1,0,1)\,,(3,1,0)\}$ for $\mathbb{R}^3$. (4)

4B) Solve $(2x+3)^2\frac{d^2y}{dx^2} - (2x+3)\frac{dy}{dx} - 12y = 6x$. (3)

4C) Find the eigen values and one of the eigen vectors of the matrix (3)

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

5A) Define minimal spanning set of vectors. Prove that a minimal spanning set of vectors forms a basis for a vector space V over F. (4)

5B) Given $\frac{dy}{dx} = 1 + xy$ with the initial condition y(0) = 1. Using Taylor series method, compute y(0.1) correct to four decimal places by considering terms up to fourth order. (3)

5C) Using the Gauss-Jordan method, find the inverse of: (3)

$$A = \begin{bmatrix} 1 & 3 & 1 \\ -1 & 1 & 2 \\ 2 & 1 & -2 \end{bmatrix}.$$

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