Question Paper

Exam Date & Time: 14-Jan-2023 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

FIRST SEMESTER B.TECH. EXAMINATIONS - JANUARY 2023
SUBJECT: MAT 1171 / MAT-1171 - ENGINEERING MATHEMATICS - I

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

1A) The area of a circle (A) corresponding diameter (D) is given below. (4)

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the area corresponding to diameter 105 using Newton's interpolation method.

1B) Solve $(1+y^2)dx = (\tan^{-1} y - x)\,dy$ (3)

1C) The following data were collected when a large oil tanker was loading. Calculate the flow rate at time $t = 20\ mins.$ (3)

t (mins)	0	10	20	30	40	50	60
V (10^6 barrels)	0.4	0.7	0.77	0.88	1.05	1.17	1.35

Also compute $\frac{d^2V}{dt^2}$ at t = 60 mins. Here, V is the volume of the oil present in the tank at time t.

2A) Using Runge Kutta method of fourth order, solve $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0) = 1$ for $y(0.2)$ taking $h = 0.1$. (4)

2B) Solve $(x^3 - 2y^2)dx + 2xydy = 0$ (3)

2C) The velocity v of a particle at distance s from a point on its path is given by the table: (3)

s ft.	0	10	20	30	40	50	60
v ft./sec.	47	58	64	65	61	52	38

Estimate the time taken to travel 60 ft by using Simpson's 1/3rd rule.

3A) Using Gauss Seidel method solve the system of equations (4)
$2x + y + 6z = 9;\ 8x + 3y + 2z = 13;\ x + 5y + z = 7.$
Carry out 4 iterations correct up to 4 decimal places

3B) Solve $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^{(e^x + 3x)}$ by the method of variation of parameters. (3)

3C) Find the root of the equation $f(x) = \cos x - xe^x = 0$ which lies between 0 and 1 by the Regula Falsi method. Carryout 4 iterations. (3)

4A) Using Gram-Schmidt orthogonalization process construct an orthonormal set of vectors (4)
from the set of $\{(1,1,1),(-1,0,-1),(-1,2,3)\}$ for $\mathbb{R}^3$.

4B) Solve $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos(\log(1+x))$. (3)

4C) Determine the largest eigen value and the corresponding eigen vector of the matrix (3)

$$A=\begin{bmatrix}2 & 0 & 1\\ 0 & 2 & 0\\ 1 & 0 & 2\end{bmatrix}$$

By Rayleigh-Power method by taking the initial vector as $[1,0,0]^T$

5A) Define the basis of a vector space. Prove that in a vector space V over a field F, every maximal (4)
linearly independent set of vectors forms a basis for V.

5B) Consider the initial value problem $y' = x(y+1), y(0) = 1$. (3)
Compute $y(0.2)$ with $h = 0.1$ using Taylor series method.
Consider up to the fourth order term.

5C) Using Gauss-Jordan method, find the inverse of $A=\begin{bmatrix}1 & 0 & 2\\ 2 & -1 & 3\\ 4 & 1 & 8\end{bmatrix}$. (3)

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