Question Paper

Exam Date & Time: 22-Dec-2022 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

THIRD SEMESTER B.TECH. (ELECTRONICS AND COMMUNICATION ENGINEERING) EXAMINATIONS - DECEMBER 2022
SUBJECT : MAT 2152 ENGINEERING MATHEMATICS III

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

1A) Expand $f(x) = x - x^2, -\pi \leq x \leq \pi,\ f(x + 2\pi) = f(x)$ as a Fourier series. (4)

1B) Find the half range sine series for the function $f(x) = x(\pi - x), 0 < x < \pi$. (3)

1C) Find the Fourier transform of $f(x) = \begin{cases} 0 & x < a \\ 1 & a < x < b \\ 0 & x > b \end{cases}$ (3)

2A) Prove that $u = e^{-x}(x\, cosy + ysiny)$ is a harmonic function. Hence find f(z) (4)

2B) If u(x, y) and v(x, y) are harmonic functions in a domain D, then prove that $\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$ is analytic in D. (3)

2C) Find Fourier cosine transform of $f(x) = (x^{a-1})$, where $0 < a < 1$. (3)

3A) Using Cauchy's residue theorem, evaluate $\int_C \frac{e^{2z}}{(z+1)(z-2)} dz$, where C is the circle $|z| = 3$. (4)

3B) Evaluate $\int_C (z + z^2)\, dz$ along the straight line from the point 1- i to 2+3i. (3)

3C) Find the Laurent's series expansion of the function $f(z) = \frac{1}{(z+1)(z+2)^2}$ in the region $2 < |z - 1| < 3$. (3)

4A) Verify Green's theorem for $\oint_C (2xy + y^2)\, dx + x^2\, dy$, where C is the boundary of the region defined by $y = x$ and $y = x^2$. (4)

4B) For any scalar function ϕ, prove that $curl(grad\ \phi) = 0$. (3)

4C) If $F = 2yi - zj + xk$, evaluate $\oint_C F\ .\ dR$ along the curve $x = cos\, t, y = sin\, t, z = 2cos\, t$ from $t = 0$ to $t = \pi/2$. (3)

5A) Solve the partial differential equation $xu_{xy} = yu_{yy} + u_y$ using the (4) transformation $v = x, z = xy$.

5B) Assuming the most general solution, Solve the heat equation $u_t = 25u_{xx}, 0 \leq x \leq 80$ subjected to $u(x,0) = sin\frac{3\pi x}{40}$ and $u(0,t) = 0, u(80,t) = 0$. (3)

5C) Use divergence theorem to evaluate $\iint_S F \cdot n\, ds$ where $F = 4xzi - y^2 j + yzk$ and S is the surface of the cube bounded by the region $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. (3)

-----End-----