Exam Date & Time: 27-Jan-2023 (09:30 AM - 12:30 PM)



MANIPAL ACADEMY OF HIGHER EDUCATION

FIRST SEMESTER B.TECH END SEMESTER EXAMINATIONS, NOV 2018 ENGINEERING MATHEMATICS III [MAT 2154]

Marks: 50

Duration: 180 mins.

(3)

A

Answer all the questions.

Instructions to Candidates: Answer ALL questions Missing data may be suitably assumed

- 1) Obtain the half range sine series expansion of f(x) = x, 0 < x < 2.
 - A) B) Expand $f(x) = 2lx - x^2, 0 \le x \le 2l; f(x + 2l) = f(x)$, as
 - C) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ Hence prove that $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$ (4)
- 2) Find the Fourier series expansion of $f(x) = \begin{cases} -1+x, & -\pi < x < 0\\ 1+x, & 0 < x < \pi \end{cases}$ (3) A) $f(x+2\pi) = f(x).$
 - B) An urn contains 5 red and 10 black balls. Eight of them are placed in another urn. What is the chance that the latter then contains 2 red and 6 black balls? (3)
 - C) The chances of A, B and C becoming the general manager of certain company are in the ratio 4:2:3. The probabilities that the bonus scheme will be introduced in the company if A, B and C become general manager are 0.3, 0.7 and 0.8 respectively. If the bonus scheme has been introduced, what is the probability that A has been appointed as general manager? (4)
- 3) Calculate the variance if the random variable X has probability density function defined

(3)

MAT 2154

(4)

A)
$$f(x) = \begin{cases} \frac{3+2x}{18}, & 2 \le x \le 4\\ 0, & Otherwise \end{cases}$$

B) Find
$$\nabla \emptyset_{if} \emptyset = ln |\vec{r}|_{where} \vec{r} = xi + yj + zk.$$
 (3)

- C) Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained and let Y be the number of queens obtained.
 - (i) Obtain the joint probability distribution of (X, Y)
 - (ii) Obtain the marginal distribution of X and Y.
 - (iii) Find the correlation coefficient ρ_{XY} .
- 4) Find the directional derivative of $\emptyset = 4xz^3 3x^2y^2z_{at}(2, -1, 2)$ in the direction 2i - 3j + 6k. (3)

A)

B) Evaluate $\iint_{S} f.n \, dS_{\text{where }} f = (x + y^2)i - 2xj + 2yzk \text{ and S is the surface of}$ the plane 2x + y + 2z = 6 in the first octant. (3)

C)
$$\frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 u}{\partial y^2} = 0, \text{ using the transformation } v = x, z = x + y.$$
(4)

5)

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A) Solve
$$\frac{\partial^3 z}{\partial x \partial y^2} + xy + \sin(2x - 3y) = 0.$$
 (3)

B)
$$\int_{C} (3x^{2} - 8y^{2})dx + (4y - 6xy)dy$$

Use Green's theorem to evaluate is the boundary of the region defined by x = 0, y = 0, x + y = 1.

C) Prove that f = (2x + yz)i + (4y + zx)j - (6z - xy)k is solenoidal as well as irrotational. Also find the scalar potential of f. (4)

where C (3)

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