Exam Date & Time: 22-Dec-2022 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

THIRD SEMESTER B.TECH END SEMESTER EXAMINATIONS, DEC 2022

ENGINEERING MATHEMATICS III [MAT 2154]

Marks: 50 **Duration: 180 mins.**

Descriptive Questions

Answer all the questions. Section Duration: 180 mins

Instructions to Candidates: Answer ALL questions Missing data may be suitably assumed

1) A) Represent the function $f(x) = x^2, -l \le x \le l, f(x+2l) = f(x)$ as a Fourier series. (3)

B) Obtain the half range cosine series expansion of $f(x) = x(\pi - x), 0 < x < \pi$. (3)

C) Find the Fourier series representation of the given function (4)

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ sinx, & 0 < x < \pi \end{cases}$$

2) A) Find the Fourier transform of $f(x) = e^{-a|x|}$, a > 0. (3)

B) If $P(A) = P(B) = P(A \cap B)$, show that $P(A \cap \bar{B}) + P(\bar{A} \cap B) = 0$ (3)

C) The contents of urns I, II and III are as follows: 1white, 2 black and 3 red marbles, 2 white, 1black and 1 red marble and 4 white, 5 black and 3 red marbles respectively. One urn is chosen at random and two marbles are drawn from it. They happen to be white and red. What is the probability that they come from urn I, urn II and urn III? (4)

3) The random variable X has the following probability function:

A) (3)

X	-2	-1	0	1	2	3
$P(x_i)$	0.1	K	0.2	2K	0.3	K

i) Find the value K

ii) Find the cumulative density function of X

iii) Find P(0< X < 3)

B) Find the scalar potential of $f = (x + 2y + 4z)i + (2x - 3y - z)j + (4x - y + 2z)k.$ (3)

C) Suppose that the two dimensional random variable (X, Y) has joint pdf

$$f(x, y) = \begin{cases} e^{-x-y}, x \geq 0, y \geq 0 \\ 0, \quad otherwise \end{cases}.$$ (4)

i) Find the marginal pdf of X and Y. (1 mark)

ii) Evaluate the coefficient of correlation ρ_{XY}. (3 marks)

4)

A) Solve $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 2(x + y)u$, by method of separation of variables. (3)

B) If $f = xyi + yzj + zxk,$ evaluate $\int_C f.dr$ where C is the curve represented by $x = t, y = t^2, z = t^3, -1 \leq t \leq 1.$ (3)

C) Derive one-dimensional wave equation with necessary assumptions. (4)

5)

A) Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$ where $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}.$ (3)

B) Find the constants 'a' and 'b' so that the surface $ax^2 - byz = (a+2)x$ is orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$. (3)

C) Verify Stoke's theorem for $F = (x^2 + y^2)i + 2xyj$ in the rectangular region $x = 0, y = 0, x = a, y = b.$ (4)

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