Question Paper

Exam Date & Time: 27-Jan-2023 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

THIRD SEMESTER B.TECH END SEMESTER MAKE UP EXAMINATIONS, JAN 2023

ENGINEERING MATHEMATICS III [MAT 2153]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions.

Instructions to Candidates: Answer ALL questions Missing data may be suitably assumed

1) A) Expand $f(x) = x - x^2, -\pi < x < \pi,\ f(x+2\pi) = f(x)$. And hence show that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots \ldots \ldots \ldots = \frac{\pi^2}{12}$. (4)

B) Obtain the half range Fourier sine series expansion of $f(x) = x, 0 < x < 2.$ Also sketch the corresponding periodic extension of *f(x).* (3)

C) Obtain the first harmonics in the Fourier series of *y,* where *y* is given by (3)

x:	0	$\frac{T}{6}$	$\frac{2T}{6}$	$\frac{3T}{6}$	$\frac{4T}{6}$	$\frac{5T}{6}$
y:	1.98	1.30	1.05	1.30	-0.88	-0.25

2) A) Find all possible series expansion of the function $f(z) = \frac{1}{z(z-1)^2}$ in the region (4)

(i) $|z-1| < 1, \quad (ii) |z-1| > 1$

B) Find an analytic function f(z), given $v = e^{-x}(y\cos y - x\sin y)$ (3)

C) $f(x) = e^{-ax}, a > 0$. (3)

Find the Fourier sine and cosine transform of

3) A) Evaluate $\int_C \frac{z+1}{z(z^2+5z+6)} dz$ where $C: |z| = 4$ using Cauchy's residue theorem. (4)

B) If $f(z) = u + iv$ is an analytic function of z=x+iy, then show that (3)

$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] |f(z)|^2 = 4|f'(z)|^2$.

C) If f(r) is a differentiable function of $r = |\vec{r}|$, then show that $\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr}$. Hence (3) find f(r) such $\nabla^2 f(r) = 0.$

4) A) Suppose a force field is given by $\vec{F} = (2x - y + z)\hat{\imath} + (x + y - z^2)\hat{\jmath} + (3x - 2y + 4z)\hat{k}.$ Find the work done in moving a particle once around a circle C in the xy-plane with its center at the origin and a radius of 3. (4)

B) Solve $u_x + u_y = 2(x + y)u,$ by separating the variables. (3)

C) Find the directional derivative of $P = 4e^{2x-y+z}$ at $(1, 1, -1)$ in a direction towards the point $(-3, 5, 6)$. (3)

5) A) Verify Green's theorem in plane for $\int_C (xy + y^2)\,dx + (x^2)\,dy$ where C is the boundary of the region defined by $y = x \text{ and } y = x^2$. (4)

B) Evaluate using Stokes theorem, $\oiint (\nabla \times \vec{A}) \cdot \hat{n}\, ds$ where $\vec{A} = (x^2 + y - 4)\hat{\imath} + 3xy\hat{\jmath} + (2xz + z^2)\hat{k}$ and S is the surface of the hemisphere $x^2 + y^2 + z^2 = 16$ above the xy-plane. (3)

C) Solve the partial differential equation $\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0$ using the transformation $v = x + t,\ \ w = x - t$. (3)

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