Exam Date & Time: 22-Dec-2022 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

THIRD SEMESTER B.TECH END SEMESTER EXAMINATIONS, DEC 2022

ENGINEERING MATHEMATICS III [MAT 2153]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions. Section Duration: 180 mins

Instructions to Candidates: Answer ALL questions Missing data may be suitably assumed

1) Expand $f(x) = \begin{cases} 2-x, & 0<x<4 \\ x-6, & 4<x<8 \end{cases}$, $f(x+8) = f(x)$. And hence show that

A) $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\frac{1}{9^2}+\cdots\ldots\ldots\ldots = \frac{\pi^2}{8}$. (4)

B) Obtain the half range Fourier cosine series expansion of $f(x) = x(l-x), 0 < x < l$. Also sketch the corresponding periodic extension of *f(x)*. (3)

C) Obtain the first harmonics in the Fourier series of *y,* where *y* is given by (3)

x:	0	1	2	3	4	5
y:	4	8	15	7	6	2

2) A) . Find all possible series expansion of the function $f(z) = \frac{1}{(z+1)(z+3)}$ in the region (i) $|z| < 1, \quad (ii) 1 < |z| < 3, \quad (iii) |z| > 3$ (4)

B) . Find an analytic function f(z), given $u - v = (x-y)(x^2+4xy+y^2)$ (3)

C) Find the Fourier transform of $f(x) = \begin{cases} 1, & |x| \le a \\ 0, & |x| > a > 0 \end{cases}$. Hence deduce that $\int_0^\infty \frac{\sin t}{t} dt = \frac{\pi}{2}$. (3)

3) Verify Cauchy's integral theorem for $\int_C z^2 dz$ where C is the triangle joining the points 0, 2 and 2i. (4)

A)

B) Evaluate $\int_C \frac{e^z}{z(z^2-3z+2)} dz$ where $C: |z| = 3$ using Cauchy's residue theorem. (3)

C) Find the acute angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at $(1, -2, 1)$. (3)

4) A) Show that $\vec{F} = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential and the work done in moving an object in this field from $(1, -2, 1)$ to $(3, 1, 4)$. (4)

B) Prove that $\vec{A} = r^n\vec{r}$ is irrotational. Find n when it is also solenoidal. (3)

C) Solve $\frac{\partial^2 u}{\partial x \partial t} = e^{-t}\cos x$ using direct integration, given that u=0 when t=0 and $\frac{\partial u}{\partial t} = 0$ when x=0. (3)

5) A) Verify Green's theorem in plane for $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region defined by $x = 0, y = 0$ and $x + y = 1$. (4)

B) Evaluate using Gauss divergence theorem, $\oiint_S \vec{A} \cdot \hat{n}\, ds$ where $\vec{A} = (2x - y)\hat{\imath} - 2y\hat{\jmath} - 4z\hat{k}$ and S is the surface of the region bounded by $x = 0, y = 0, z = 0, z = 3$ and $x^2 + y^2 = 16$ lying in the first octant. (3)

C) . Solve the partial differential equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$ using the transformation $v = x + ct, w = x - ct$ subject to the conditions $u(x, 0) = \varphi(x)$ and $u_t(x, 0) = \psi(x)$. (3)

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