Question Paper

Exam Date & Time: 22-Dec-2022 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

THIRD SEMESTER B.TECH. (COMPUTER SCIENCE & ENGINEERING / INFORMATION TECHNOLOGY) EXAMINATIONS - DECEMBER 2022
SUBJECT : MAT 2155 - ENGINEERING MATHEMATICS III

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

1A) Find both the 114th and the 78th permutations of 1, 2, 3, 4, 5 in both lexicographical order and Fike's order. (4)

1B) If repetition is not allowed, how many four digit numbers can be formed from the digits 1, 2, 3, 5, 7, 8. How many of the numbers are lesser than 4000? How many are divisible by 4? (3)

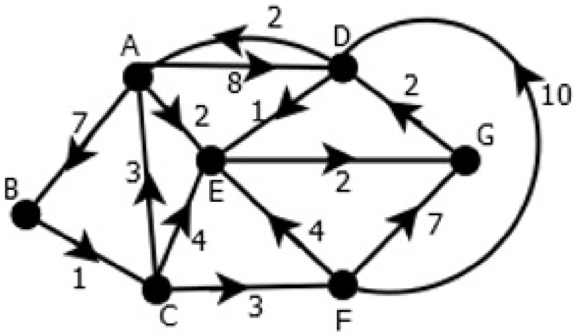
1C) Let $(A, \vee, \wedge, -)$ be a finite Boolean algebra. Let b be any non-zero element in A and $a_1, a_2, a_3, \ldots, a_k$ be all the atoms satisfying $a_i \leq b, i = 1, 2, 3, \ldots, k$. Prove that $b = a_1 \vee a_2 \vee a_3 \vee \ldots \vee a_k$. (3)

2A) (4)
i) Define $*$ on Q^+ as $a * b = \frac{ab}{4}$, for all $a, b \in Q^+$, where Q^+ is a set of positive rational numbers. Then verify whether $(Q^+, *)$ is a group?
ii) Show that any group with at most five elements is abelian.

2B) Prove that a nontrivial connected graph G is Eulerian if and only if every vertex of G has even degree. (3)

2C) Show that if a lattice is distributive then for elements a, b, c (3)
$(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$

3A) Using Dijkstra's algorithm, find the shortest path from F to all other vertices for the network given below. (4)



3B) How many ways are there to distribute 25 identical balls into seven distinct boxes if the first box can have no more than 10 balls but any number can go into each of the other six boxes? (3)

3C) If G is a connected graph, such that diameter of $G \geq 3$, then diameter of $\overline{G} \leq 3$. (3)

4A) Use generating function, calculate the number of ways in which can an examiner assign 30 marks to 8 questions so that no question receives less than 2 marks? (4)

4B) Let $E(x_1, x_2, x_3) = (\overline{x_1} \wedge x_2 \wedge \overline{x_3}) \vee (x_1 \vee \overline{x_2}) \vee (x_1 \vee x_3)$ be a Boolean expression over the two valued Boolean algebra $\{0, 1\}$. Express $E(x_1, x_2, x_3)$ as a both disjunctive and conjunctive (3) normal form.

4C) In a lattice (A, ≤), for any elements $a, b, c \in A$ show that if $a \leq b$ then $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$. (3)

5A) (i) Let H and K be subgroups of a group G. Show that H ∩ K is a subgroup of G. Is $H \cup K$ a subgroup of G? (4)

(ii) Prove that any group of prime order is cyclic.

5B) Show that $r \wedge (p \vee q)$ is a valid conclusion from the premises $p \vee q, q \rightarrow r, p \rightarrow m, \neg m$. (3)

5C) Show that $(\exists x) M(x)$ follows logically from the premises $(\forall x)[H(x) \rightarrow M(x)]$ and $(\exists x) H(x)$. (3)

-----End-----